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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 1, Number 3, May 1979

0002-9904/79/0000-0215/$01.50

Algebraic methods in the global theory of complex spaces, by Constantin Bănică and Octavian Stănăsilă, J. Wiley, London, New York, Sydney, Toronto, 1976, 296 pp., $21.50. Revised English version of original Romanian text published in 1974.

The algebraic methods referred to in the title are primarily those of sheaves and cohomology. This book presents an account, with complete proofs, of recent advances in the theory of complex analytic spaces.

Complex analytic spaces are complex manifolds with singularities based on the local model of zeros of a finite number of holomorphic functions in an open set in $\mathbf{C}^n$ (an analytic variety). A sheaf over a space is essentially an assignment to each point of the space of local data near that point (e.g. holomorphic functions near a point, holomorphic functions which vanish on a given subvariety, etc.). Sheaf cohomology is an obstruction theory which encodes information on passing from the solution of certain local problems to certain global problems. For instance, a subvariety is defined as a closed set which is locally the zeros of holomorphic functions, and one can ask whether this same subvariety is the zero set of globally defined holomorphic functions. The use of sheaves and cohomology in the study of complex spaces has been extremely fruitful since the initial work of Cartan and Serre in this direction in the 1950s.

The statements of results in this book are closely parallel to the analogous results in algebraic geometry, mostly due to Grothendieck, though the proofs are different, and usually more difficult than in the algebraic case. This approach should appeal to algebraic geometers wishing to learn about complex analytic spaces, since it highlights the similarities between the two theories. It should also be useful to specialists in several complex variables, since it assembles very recent and widely scattered material. There is no index, but there is an extensive bibliography, in which 3/4 of the articles have appeared since 1960.

The idea of using sheaves and cohomology in the study of several complex variables is not new. Sheaves were introduced by Leray (1950) to study the cohomology of fibre spaces. One of their first serious applications was by H. Cartan (with the assistance of Serre), in his Paris seminars of 1951/1952 and 1953/1954, where he used them to reinterpret and expand the work of Oka, thus laying new foundations for the theory of several complex variables. Grauert and Remmert built upon this foundation with a series of papers, including the development of the concept of a complex analytic space.

Meanwhile Serre (1955) introduced these same techniques of sheaves and cohomology into algebraic geometry, where they thrived in the hands of Grothendieck, who developed many new techniques, and proved theorems at first parallel to those in several complex variables, but soon outdistancing them. Thus, for example, the theories of local cohomology, formal completions, and duality for a proper morphism appeared first in the algebraic case.

These algebraic results in turn provided a model for analogous developments in the complex-analytic case, which have been accomplished in the last ten to fifteen years. This book is an account of that work. Some of the landmarks are the coherence of direct image sheaves, due to Grauert (1960); the general duality theorems, due to Ramis, Ruget, Verdier, and Suominen (1968–1971); and the theory of local cohomology and gap-sheaves of Siu and Trautmann (1971). A recent result, due to the first author, is the compatibility of formal completion with direct image of coherent sheaves by a proper morphism. All of these are treated here. In each case some analysis is necessary to get started, but once one has sufficient foundations, one can apply the same algebraic machinery as in the case of schemes.

The strength of this book is that it assembles these results into one coherent exposition. Its weakness is insufficient motivation and applications. It is a technical book, without general discussions, without exercises, and without applications. For example, the technical methods of local cohomology, gap-sheaves, and depth are treated in considerable detail, but without giving their main application, which is to the problem of extension of coherent analytic sheaves, raised by Serre (1966), and solved by Siu and Trautmann (1971).

I have the impression that some results are included here more for the sake of imitating the algebraic case than for their possible use in applications. This is particularly evident in the section on projective morphisms, where, given the techniques already developed, the proof of these theorems in the complex-analytic case is hardly more than an exercise. To do something more substantial, one should cast these results in their natural degree of generality, which would be a projective morphism of quite general ringed spaces. Grothendieck already pointed out (in the Cartan seminar of 1960/1961) the necessity of developing a theory of relative analytic spaces over very general base spaces (e.g. differentiable manifolds). This is because in questions of deformations and moduli (not mentioned in this book) the parameter spaces may not be complex-analytic. Recently several authors have developed theories of relative analytic spaces, and have proved Grauert's coherence and semicontinuity theorems in this context, generalizing some old results of Kodaira and Spencer: Forster and Knorr (1972), Kiehl (1972), Schneider (1972), Houzel (1973). It is a pity that the authors of this book did not take the opportunity to put their results in a suitably general context, or at least include some discussion of this line of work and its applications.

In spite of these shortcomings, this will be a valuable book for anyone interested in the most recent techniques in the study of complex analytic spaces.

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