integral should at the very least enter into a careful discussion of why the Bochner integral does *not* mimic the Lebesgue integral with regards to the fundamental theorem of calculus. Included in such a presentation ought to be the classical results of a positive nature regarding the differentiation of vector-valued absolutely continuous functions. Many other things might be equally important for application sake, but before we get carried away with this line of thought let it be said once and for all that the book under review (no I've not forgotten it!) is *not* about the Bochner integral! I know the title says it is and many of its results pertain to the Bochner integral but the Bochner integral is not the real subject of the book.

To paraphrase Professor Mikusiński slightly, the purpose of the book is to give an approach to the theory of the *Lebesgue* integral which would be "as intelligible and lucid as possible–understandable to students in their first undergraduate courses."

The aim is laudable and, taken with a touch of reason, it is achieved. The inclusion of the Bochner integral is due to the fact that Professor Mikusiński's approach to the Lebesgue integral "extends automatically to the Bochner integral (by replacing real coefficients of series by elements of a Banach space)." To be sure, the enthusiasm expressed in the above paragraph of making the material available to first year undergraduates is excessive but with that as a goal, Professor Mikusiński attains an interesting and readable introduction to the Lebesgue integral. The book under review suffers from several glaring omissions. Most notable absence–exercises! Professor Mikusiński's approach to the Lebesgue integral is not standard (though not as novel as one might conclude from the text) and, therefore, exercises might well serve to buoy the confidence of the reader. Standard topics such as the Lebesgue spaces and Fourier series are not to be found in this book. On the other hand, Professor Mikusiński's treatment of "Changes of Variables" is noteworthy. Also, the fact that he does present the Bochner integral with so little extra work might be considered a plus for the book. Since so little extra work is needed, the failure to discuss the validity of a general fundamental theorem is an "opportunity lost."

In summary, the title of the book is misleading but the contents worthwhile. It might be that *An approach to Lebesgue integration* or even *Lebesgue integration made simple* would be appropriate as a title of the book and descriptive of the aims of the author. *The Bochner Integral* seems inappropriate in each role.

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*A course in mathematical logic*, by Yu. I. Manin, Graduate Texts in Math. vol. 53, Springer-Verlag, New York, Heidelberg, Berlin, 1977, xiii + 286 pp., $19.80.

The principal aim of a graduate textbook on logic should, I think, be to enable the reader to understand the current literature in the subject. What material would such a book have to cover?

First of all, of course, comes the fundamental syntax and semantics of first-order logic. The mathematics here is not very profound; the Completeness Theorem is the only deep result. The important point is to familiarize the reader with those techniques which are special to logic. He must become accustomed to dealing with symbols and expressions as mathematical objects. He must understand the difference between semantics and syntax and the relation between them. Above all, he must understand what can be said in first-order logic; e.g., he must understand why in doing first-order topology, it is the subsets of the topological space (rather than the points of the space) which must be taken as individuals.

After this elementary part, the material branches into the fundamental fields of Model Theory, Recursion Theory, Set Theory, and Proof Theory. We consider these separately.

Model Theory seems to immediately split into a number of subfields.

(1) The Compactness Theorem and its Consequences. The material here is fairly elementary, and should be discussed (but at not too great length).

(2) Preservation Theorems. These relate the syntactical form of the axioms of a theory to closure properties of the set of models of the theory. Since this field is now quiescent, a brief introduction should suffice.

(3) Cardinals of Models. The fundamental results of Löwenheim-Skolem-Tarski can be quickly treated. Much deeper are the two-cardinal theorems (which also consider the cardinality of definable subsets of the model). One could only try to give an introduction to these.

(4) Saturation Theory. Roughly, a saturated model is one containing as many kinds of objects as possible. There are elegant elementary results, deep results, and many open problems here. A good introduction to both this and two-cardinal theorems would be given by a proof of Morley's Categoricity Theorem.

(5) More general logics. The chief results here concern infinite conjunctions and disjunctions and generalized quantifiers. This is a very active field, and there should be some introduction to it.

(6) Soft logic. This is the study of logics in general. The most important theorem, due to Lindstrom, characterizes first-order logic; it is elegant and not too difficult.

In Recursion Theory, one first needs an introduction to recursive functions (including, in fact emphasizing, partial functions) and recursively enumerable sets. The most important applications are the Imcompleteness Theorem and results of Church and Tarski on undecidability of theories. Next, one should have an introduction to the arithmetical and analytical hierarchies, including at least the Souslin-Kleene theorem. There are many further subjects: degree theory, higher type recursive functions, recursive functions on sets and ordinals, etc. Fortunately, these can be omitted or treated briefly.

In set theory, one must first introduce the axioms of ZFC, explain where they come from, and show how they can be used to develop ordinals and cardinals. After that, there is an embarrassment of riches. One would like to say something about constructible sets, forcing, descriptive set theory, large cardinals, and (perhaps) determinacy. Each of these topics could easily fill a book; but one could hope to intoduce each of them and include at least one nontrivial result for each.

Proof Theory has largely gone its own way, and can be treated briefly in a general logic text. Most important is the Gentzen method of elimination of cuts, which should be discussed at least briefly.

Finally, there are subjects which cut across these fields. Most notable is the theory of admissible sets, which is both a unifying idea and a valuable tool. One would hope to include an introduction to this topic.

How well does Manin's book cover this material? The basic material on first-order logic is there and is done well. The only result in Model Theory is the Löwenheim-Skolem Theorem. In Recursion Theory, he covers the material mentioned above through the arithmetical hierarchy, but with some essential results (like the Enumeration Theorem for partial recursive functions) missing. In set theory, constructible sets and forcing (in the Boolean model form) are treated fairly extensively, but the other advanced topics are not mentioned. There is no Proof Theory and no mention of admissible sets.

On the other hand, Manin has proved several significant results not in the above list, e.g., Higman's Theorem on embeddings in finitely presented groups and the Kochen-Specker results on quantum logics. There is no doubt that these results and their proofs are interesting; but the techniques are special to the problem and not of much general use.

After this long discussion on content, a word about style. The book meets reasonable standards of clarity and elegance; but it's outstanding feature is its liveliness. Manin is interested in everything; and there are many (perhaps too many) asides on topics connected in some way with logic. No one should be bored by this book.

A lively book treating the fundamentals of logic and some important advanced topics–surely this is enough to make the book worthwhile. Still, I cannot help looking forward to the book which will treat most of the topics above in an equally lively way. It will surely be the logic textbook for the 1980s.

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*Saks spaces and applications to functional analysis*, by J. B. Cooper, Mathematics Studies, Vol. 28, North-Holland, Amsterdam, New York, Oxford, 1978, x + 325 pp.

The term "Saks space" has no fixed meaning in the literature, but it always refers to some variant of the following situation: a normed vector space $(E, \| \|)$ provided with a distance function or a topology $\tau$. Depending on the author, a Saks space is either a triple $(E, \| \|, \tau)$ or a pair $(B, \tau_B)$ where $B$ is the unit ball $\{x \in E: \|x\| \leqslant 1\}$ and $\tau_B$ is the induced metric or topology. The conditions imposed on $E$, $\| \|$ and $\tau$ may vary; usually it is assumed that

(*) $\tau$ is coarser on $E$ than the norm topology and $B$ is closed in $(E, \tau)$, together with further restrictions like metric completeness of $(B, \tau_B)$ or local