

the book is a thorough compilation of what was known up to that time about the Basic Problem for (4).

Since the book was written there has been considerable activity in equations of variable type, especially in the Soviet Union. As one might expect in a newly developing field, many of the results are fragmentary. For example, a spate of papers has dealt with equations with discontinuous coefficients. Other papers have developed properties of solutions of variable equations of order higher than two. Still a third group of results is based on the equations of gas dynamics which are of mixed type when the flow has both supersonic and subsonic regions.

It seems likely that the bits and pieces of all these results will be put together eventually to form a single comprehensive theory for mixed type equations in  $R^2$  similar to the classical theory for the standard elliptic, hyperbolic and parabolic equations. At this time the task appears formidable; the development in three and more dimensions and for equations of order other than two is even more remote.

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*The Bochner integral*, by Jan Mikusiński, Academic Press, New York, 1978, xii + 233 pp.

In the 1930's a great effort was made to develop the basic theory of Banach space valued functions of a real variable. The pioneers in this study (Bochner, Dunford, Gelfand, Pettis, Phillips and Richart) developed a number of integrals of varying strengths for a multitude of purposes: oftentimes, the representation of operators on concrete spaces was the object; quite as often a desire just to understand the abstract process of integration was sufficient motivation. Of the integrals developed, one integral, the Bochner integral, emerged as the strongest and, to-date, it is the Bochner integral that has been the most useful.

Curiously, the Bochner integral is the easiest of the vector integrals from yesteryear to develop and the one with the most transparent structure. Indeed, most of the usual results valid for the Lebesgue integral easily adapt to the Bochner setting. One notable exception: the fundamental theorem of calculus for absolutely continuous functions defined on  $[0, 1]$ . It is simply not the case that an absolutely continuous vector-valued function defined on  $[0, 1]$  need be the indefinite Bochner integral of its derivative—at least not unless the vector values are suitable chosen. This pathology is not all together a bad thing. The study of the class of Banach spaces for which the fundamental theorem remains valid has kept a number of mathematicians busy and off the streets for the past five years at least. This class of spaces (whose members answer to the name “Radon-Nikodym”) has come to play an important role in modern Banach space theory especially as it interacts (and it does so quite nicely) with probability theory, harmonic analysis and the infinite dimensional topology. Any book purporting to be about the Bochner

integral should at the very least enter into a careful discussion of why the Bochner integral does *not* mimic the Lebesgue integral with regards to the fundamental theorem of calculus. Included in such a presentation ought to be the classical results of a positive nature regarding the differentiation of vector-valued absolutely continuous functions. Many other things might be equally important for application sake, but before we get carried away with this line of thought let it be said once and for all that the book under review (no I've not forgotten it!) is *not* about the Bochner integral! I know the title says it is and many of its results pertain to the Bochner integral but the Bochner integral is not the real subject of the book.

To paraphrase Professor Mikusiński slightly, the purpose of the book is to give an approach to the theory of the *Lebesgue* integral which would be "as intelligible and lucid as possible—understandable to students in their first undergraduate courses."

The aim is laudable and, taken with a touch of reason, it is achieved. The inclusion of the Bochner integral is due to the fact that Professor Mikusiński's approach to the Lebesgue integral "extends automatically to the Bochner integral (by replacing real coefficients of series by elements of a Banach space)." To be sure, the enthusiasm expressed in the above paragraph of making the material available to first year undergraduates is excessive but with that as a goal, Professor Mikusiński attains an interesting and readable introduction to the Lebesgue integral. The book under review suffers from several glaring omissions. Most notable absence—exercises! Professor Mikusiński's approach to the Lebesgue integral is not standard (though not as novel as one might conclude from the text) and, therefore, exercises might well serve to buoy the confidence of the reader. Standard topics such as the Lebesgue spaces and Fourier series are not to be found in this book. On the other hand, Professor Mikusiński's treatment of "Changes of Variables" is noteworthy. Also, the fact that he does present the Bochner integral with so little extra work might be considered a plus for the book. Since so little extra work is needed, the failure to discuss the validity of a general fundamental theorem is an "opportunity lost."

In summary, the title of the book is misleading but the contents worthwhile. It might be that *An approach to Lebesgue integration* or even *Lebesgue integration made simple* would be appropriate as a title of the book and descriptive of the aims of the author. *The Bochner Integral* seems inappropriate in each role.

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*A course in mathematical logic*, by Yu. I. Manin, Graduate Texts in Math. vol. 53, Springer-Verlag, New York, Heidelberg, Berlin, 1977, xiii + 286 pp., \$19.80.

The principal aim of a graduate textbook on logic should, I think, be to enable the reader to understand the current literature in the subject. What material would such a book have to cover?