Despite the high quality of research material, the book at times suffers from some lapses. For example, a reader will often come across sentences like "... admits completing web which elements are star-shaped..." (p. 49, Proposition IV). Certainly an English reader would prefer "... the elements of which..." instead of "... which elements...". Also some brevities (intentional or otherwise) are awkward e.g. the frequent usage of "T is continuous of E into F" will be more acceptable to an English-hearing ear if it is either "T is a continuous map of E into F" or "T is continuous from E into F". Also, the year of reference [30] is incorrect. Nonetheless, the monograph is a welcome addition to the mathematics literature.

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Solutions of ill posed problems, by A. N. Tikhonov and V. Y. Arsenin (F. John, Translation Editor) V. H. Winston and Sons (distributed by Wiley, New York), 1977, xiii + 258 pp., $ 19.75.

The mathematical investigation of ill posed or improperly posed problems in mathematics, mathematical physics and engineering is probably one of the least understood and most misunderstood and maligned endeavors of our science. Ever since the appearance of Hadamard's famous example of a nonwell posed problem [4], the Cauchy problem for the Laplace equation, some mathematicians have regarded the study of such problems as a waste of intellectual effort. There are two or three reasons for this attitude. First of all, what possible physical interest could there be in problems for which existence, uniqueness or perhaps continuous dependence failed? Hadamard himself remarked "But it is remarkable, on the other hand, that a sure guide is found in physical interpretation: an analytic problem always being correctly set... when it is the translation of some mechanical or physical question." Another justification for the lack of interest in such problems is given by Courant [2]. "Unfortunately little mathematical progress has been made in the important task of solving or even identifying such problems . . . but [they] are still important and motivated by physical considerations." He then goes on to restrict his attention to the classical well posed problems. Finally, the proposition has been put forth by several well-known workers in partial differential equations and continuum mechanics that the whole field is primarily concerned with trying to prove uniqueness and continuous dependence results for initial-boundary value problems for partial differential equations of mixed or indeterminant type. None of these reasons has much merit today, if indeed they ever did. Courant himself gave several important and interesting physical and mathematical problems that are not well posed. Moreover, in the last thirty or forty years there has been a tremendous upsurge of interest in and progress made on such problems, not only because they not only occur naturally in the mathematical formulation of physical problems, but because they arise in partial differential equations, control theory, linear programming, integral equations, continuum mechanics, function theory, Fourier analysis and so on. Several areas of the applications have been sources for such problems including geophysics, holography, radio physics, rocketry, antenna design, stellar dynamics, etc. L. E. Payne [6], [7], M. M. Lavrentiev [5] and the authors of the present volume have given rather complete bibliographies which should serve not only to indicate that the field of ill posed problems includes problems from a wide variety of scientific and mathematical disciplines but also that there are today many sophisticated and ingenious techniques for their identification and solution. In [6] one can also find a very lucid and comprehensive overview of the subject as it exists today.

Before discussing the contents of the book, which we shall do in only the barest detail, it might be helpful to explain in more precise terms to the reader exactly what is meant by a nonwell posed problem and to provide an interesting example of such a problem. Suppose (Z, ρ_z) and (U, ρ_u) are metric spaces and $A: Z \to U$ is some function from Z to U. Then the problem: Given $u \in U$, find $z \in Z$ such that $Az = u$, is said to be well posed if A has a continuous inverse $A^{-1}: U \to Z$. That is, (i) for $u \in U$ there is $z_u \in Z$ such that $Az_u = u$, (ii) z_u is unique, (iii) for all $\varepsilon > 0$ there is $\delta(\varepsilon, u) > 0$ such that $\rho_u(u, u_1) < \delta$ implies $\rho_z(z_u, z_{u_1}) < \varepsilon$. A problem is not well posed if (i), (ii) or (iii) fail. This is of course a precise, but deceptively simple definition. If one thinks of Z and U as being spaces of functions and A some complicated, perhaps even nonlinear, differential or integral operator, the apparent simplicity then vanishes and we are left with the uneasy, if correct, feeling that each problem must be considered to a greater or lesser degree on its own merits. Speaking again in only the most general terms, a problem may fail to be well posed for several reasons. Perhaps the space U is too large or Z too small. (This suggests that one ought to reformulate his problem in a weaker setting or that the problem is under or over determined with too few or too many side conditions. The latter would lead to nonexistence while the former to nonuniqueness of solutions.) Perhaps the choice of metrics was not correct and the ρ_u metric should be stronger (while perhaps simultaneously decreasing the class U) or the ρ_z metric weakened (while increasing the class Z of admissible solutions at the same time). It may be that the operator A is not the one which best describes the physical situation and should perhaps be replaced by a one which, although perhaps more complicated, does. (This is sometimes called regularization. It could even involve replacing a linear operator by a nonlinear operator.) It may even be that the problem $Az = u$ never has a solution. Then one would have to content oneself with so called quasi solutions, that is elements $z \in Z$ such that $\rho_u(Az, u)$ is as small as possible. We shall content ourselves with a single well-known example, that of the Dirichlet problem for the vibrating string to illustrate the above framework. (See [1], [3] for the details.) Suppose one has a string of length L fixed at both ends and that it is vibrating in some manner. We photograph the shape of the string at two different times, say $t = 0$ and $t = T$. Question: What would the shape of the string look like if we photographed it at some intermediate time t_1, t_1 arbitrary? Let us formulate the problem mathematically: Let $C_0^1[0, L] = \{f\colon [0, L] \to R \mid f' \text{ exists, is continuous on } (0, L) \text{ and } f(0) = f(L) = 0\}$. Let $U = C_0^1(0, L) \times C_0^1(0, L)$ and let Z denote the set of real valued functions $z(x, t)$ defined on the rectangle $[0, L] \times [0, T]$, continuous in the closed rectangle, twice continuously differentiable in the open rectangle, is such that $\partial^2 z/\partial t^2 - \partial^2 z/\partial x^2 = 0$ and $(z(\cdot, 0), z(\cdot, T)) \in U$. The operator $A\colon Z \to U$ may be described as $Az = (z(\cdot, 0), z(\cdot, T))$ and the problem is, given $(f, g) \in U$, find $z \in Z$ such that $Az = (f, g)$. This is well known *not* to be a well posed problem. In fact $(f, g) = (0, 0)$ does not imply $z \equiv 0$ unless $T/\pi L$ is irrational. Moreover, the question of existence is harder. It can be shown [1] that existence holds if $T/\pi L$ is an algebraic number. A somewhat less restrictive condition can be given for existence of weak solutions [3].

One could raise several objections to this model. After all, the wave equation is only an approximation to the dynamics of the motion for small deflections of the string. Also, if one photographs a string in motion, one will obtain a somewhat blurred image no matter how small the exposure time. So perhaps the correct problem would be to specify the solution over $[0, L] \times [0, \delta]$ and $[0, L] \times [T - \delta, T]$. However this problem is also not well posed, this time because it is overdetermined. Knowledge of z over $[0, L] \times [0, \delta]$ implies knowledge of $\partial z/\partial t(\cdot, \delta)$ and then z is forced upon us up to $t = T$ by the classical results for the Cauchy problem for the wave equation.

The current volume begins with a discussion of some examples of nonwell posed problems in integral equations and continues with an extension of the abstract notions mentioned above such as quasi solutions and regularization. Several examples are given. There follow chapters on singular and ill conditioned systems of linear algebraic equations, approximate solutions of integral equations, regularizing methods for integral equations, stable methods for summing Fourier Series, stable methods of minimizing functionals and for solving optimal control problems and for optimal planning (linear programming) problems. There is an excellent bibliography of the Russian literature in the area which complements that of Payne [6]. Regretfully, relatively few examples from partial differential equations are included, but this perhaps reflects the reviewer's bias more than anything else.

The reviewer believes that this book represents a fine contribution to the current literature on the subject of ill posed problems. The material should be of interest to a wide variety of specialists in applied mathematics and engineering and should be on the bookshelf of anyone interested in ill posed problems. There are numerous examples and illustrations. The translator has taken pains to insure that the English reads smoothly.

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Catastrophe theory and its applications, by Tim Poston and Ian Stewart, Surveys and Reference Works in Mathematics, Pitman, London, 1978, xviii + 491 pp., $50.00.

What mathematical discovery has provoked recent articles in *Scientific American*, *Nature*, *Newsweek*, *Science*, *The New York Times*, *The Times Higher Education Supplement*, *L'Express* and the *New York Review of Books*? What current theory now ranks only behind the weather and old movies as a subject of cocktail conversation between mathematicians and nonmathematicians? Is there any content to this theory which has been described in *Science* as an emperor without clothes? Has all the notoriety been public relations–beginning with the creator's brilliant choice of name? This, for instance, has led *The New York Times* to blunder on its front page article with the headline "Experts Debate the Prediction of Disasters." In short, is this theory really–as *Newsweek* described it–the most important mathematical advance since Newton's invention of the calculus?

The answer to the last question is simply no; however, catastrophe theory does have merit both in mathematics and in applications. How, then does one find out about its successes and why is there a controversy? The answers to these questions are related, but before discussing them one point should be made. To my knowledge no one has suggested that the mathematics behind catastrophe theory is anything less than superb.