following Cartan. This comparison is somewhat tricky, though elementary, and would be easier to follow if illustrated by examples (but none are given).

Finally, Chapter 8 carries out the classification (due to Cartan, with improvements by F. Gantmacher and others) of real forms of a complex semisimple Lie algebra. The chapter begins with the determination of maximal subalgebras of maximal rank in a compact Lie algebra, following A. Borel, J. de Siebenthal. Here, as elsewhere in the book, there is some lack of connecting narrative; in particular, the reader is not told right away why these subalgebras are being investigated. Again some crucial use is made of results from (6.9), but otherwise the exposition is self-contained.

This outline indicates both some strengths and some weaknesses in the authors' approach. Some of the topics they cover (affine Weyl groups, real representations, classification of real forms) are not so often treated in other standard books, so the researcher in Lie theory may find this book quite helpful as a reference. But for Lie algebras, Samelson [5] probably provides a better first course, and for Lie groups, Varadarajan [6] tells the story in a more organized, self-contained way. (For an approach emphasizing linear Lie groups, [2] would also be well worth considering if it were written in something closer to English.) The ultimate book has not been–and doubtless will not be–written. Goto and Grosshans have given a generally clear account of semisimple Lie algebras in the spirit of Cartan and Weyl, but they have not reached a fully satisfactory compromise in their parallel treatment of Lie groups.

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Closed graph theorems and webbed spaces, by M. De Wilde, Research Notes in Mathematics, no. 19, Pitman, London, 1978, xii + 158 pp.

It was in the early thirties, during the depression, that the twins (if I may call them so) of Functional Analysis were born, thanks to Banach [**1**]. I mean, of course, the open mapping (o.m.) and closed graph (c.g.) theorems: Let E, F be two complete metrizable topological vector spaces (called F-spaces for short). Then (o.m.): each linear continuous map of E onto F is open; (c.g.): each linear map of F into E with closed graph is continuous. (These are two of the basic theorems one learns in a graduate course on functional analysis.) Immediately after their birth, the twins prominently figured in the centre-fold of applications in Analysis. For example, by the help of the c.g. theorem above, one proves very quickly that the projection determined by a closed subspace of a Hilbert space is a bounded or continuous operator. The fact that the coordinate functionals determined by a basis in an F-space are continuous is a consequence of the o.m. theorem above. (A sequence $\{x_n\}$ in a topological vector space E is said to form a basis if for each $x \in E$ there exists a unique sequence $\{\alpha_n\}$ of scalars such that $x = \Sigma\alpha_n x_n = \lim_{n\to\infty} \Sigma_{k=1}^n \alpha_k x_k$ and each $\alpha_n(x) = \alpha_n$ is called a coordinate functional.) When one shows that the dual space of an L_p-space ($1 \leqslant p < \infty$) can be identified with L_q ($1 < q \leqslant \infty$; $1/p + 1/q = 1$), one calls upon the o.m. theorem for help. And when one shows that the norm topology of a commutative semisimple Banach algebra is unique, one has the backing of the c.g. theorem. These are just a few examples to indicate the depth and the power of the twins.

During the thirties and forties when the applications of the twins were rampant, the so-called Bourbakian vogue was permeating the mathematical thoughts. One of the themes of the Bourbakian philosophy is to find spaces for which a known theorem can be carried over. Naturally, the twins among others were also affected by this theme. Thus came o.m. and c.g. theorems due to Dieudonné and Schwartz [2] for LF-spaces (i.e., those locally convex spaces which are strict inductive limits of sequences of locally convex F-spaces) and subsequently Köthe [6] proved the twins for generalized LF-spaces as well. In the early fifties Grothendieck [3] proved them for more general spaces which include those mentioned above. Specifically he replaced E by a generalized LF-space and F by a β-space (i.e., the inductive limit of Banach spaces) in the theorems above. The β-spaces were later called ultrabornological. For many application purposes Grothendieck's results were more then adequate. But, still, a question was asked if one could improve the range space E while keeping the domain space F to be ultrabornological in the Grothendieck c.g. theorem. The answer was given in the sixties by the author [9] of this book where he succeeded in replacing the range space by webbed spaces. In the meantime, other developments also occurred.

As one knows that each ultrabornological space as well as each LF-space is a barrelled space and so is a Baire locally convex space, the guardians of the twins looked at barrelled spaces with expectations and out came the Pták's theory of B-complete spaces [7] which was subsequently generalized by this reviewer (c f. for instance [4]) and extended even to topological groups [5]. Pták showed that the o.m. theorem is valid if E is B-complete and F barrelled. Later in the fifties, A. Robertson and W. Robertson [8] showed that even the c.g. theorem is valid for the same pair. So the twins matured but in asymmetric fashion–asymmetric in the sense that the domain and range spaces of the twins do not belong to the same class as is the case in the original version due to Banach [1], where E, F both are F-spaces. As a matter of fact, the generalizations of the domain and the range spaces in the twins became a playful see-saw game as is clear from the existing literature. In spite of the asymmetry, one might say the twins grew to be Siamese twins. The development of some such generalizations is recorded in this reviewer's book [4], whereas this book deals with the development of the twins following the Grothendieck's generalization as mentioned above. By means of webbed space, the author generalizes the Grothendieck's theorem and supplies with numerous examples including Schwartz spaces and the inductive limits of certain Banach spaces. In addition, the author proves a so-called Schwartz closed graph theorem where one replaces the topological property of being closed on the graph by a measure-theoretic one. The reader will find the book arranged as follows:

In Chapter I, the author recalls the background material by giving the classical o.m. and c.g. theorems including the Grothendieck's results. In Chapter II, he explains why he confines to ultrabornological spaces instead of barrelled spaces by claiming that the "gap" between the two classes is "small"–a disputable statement since one can easily disagree with his "smallness" criterion. In my judgement, one can easily claim that the natural class to which the range space belongs for an o.m. theorem is that of barrelled spaces or even quasibarrelled spaces. Anyway, I think both approaches are desirable without any specific justification. The author analyzes the Grothendieck's result and others and is led to the introduction of webbed spaces which he takes up in Chapter III. To comprehend a webbed space, the reader is invited to imagine a sequence of coverings of a linear space and then take the family of finite intersections of elements from these coverings. This family is said to form a web. A linear space with a web is called a webbed space. Chapter IV is devoted to the study of such spaces in details. It turns out that this class has remarkable permanence properties and includes a lot of spaces used in Analysis for application purposes. A general c.g. theorem is proved: Let F be an ultrabornological space, E a webbed space and R a relation of F into E with the fast sequentially closed graph; then R is continuous. Moreover, if R is a relation of a locally convex space F into another locally convex space E such that the domain of R in F is of second category and the graph of R is webbed, then R is continuous. Obviously the reader can detect resemblance with the classical Banach's c.g. theorem [1] for linear maps mentioned above. In Chapter V, the author is naturally led to the study of webs in the space of linear maps from one linear space E to another linear space F. This consideration supplies us with a webbing criterion for tensor products. In discussing the linear maps from one topological vector space into another, the author takes the reader into Chapter VI to prove an o.m. theorem which he calls a homomorphism theorem and gives several equivalent statements of it. The monograph technically ends with Chapter VII containing certain properties of Suslin spaces that are relevant for the Schwartz closed graph theorem which he extends to certain topological groups. The bibliography (which is not exhaustive) followed by a short index completes the book.

The monograph is certainly written in a scholarly style and is useful to research workers and advanced graduate students. Putting together this monograph with the reviewer's monograph [4], the reader will find a reasonably complete picture of the development made in understanding the twins and their generalizations since their inception.

Despite the high quality of research material, the book at times suffers from some lapses. For example, a reader will often come across sentences like "... admits completing web which elements are star-shaped..." (p. 49, Proposition IV). Certainly an English reader would prefer "... the elements of which..." instead of "... which elements...". Also some brevities (intentional or otherwise) are awkward e.g. the frequent usage of "T is continuous of E into F" will be more acceptable to an English-hearing ear if it is either "T is a continuous map of E into F" or "T is continuous from E into F". Also, the year of reference [30] is incorrect. Nonetheless, the monograph is a welcome addition to the mathematics literature.

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Solutions of ill posed problems, by A. N. Tikhonov and V. Y. Arsenin (F. John, Translation Editor) V. H. Winston and Sons (distributed by Wiley, New York), 1977, xiii + 258 pp., $ 19.75.

The mathematical investigation of ill posed or improperly posed problems in mathematics, mathematical physics and engineering is probably one of the least understood and most misunderstood and maligned endeavors of our science. Ever since the appearance of Hadamard's famous example of a nonwell posed problem [4], the Cauchy problem for the Laplace equation, some mathematicians have regarded the study of such problems as a waste of intellectual effort. There are two or three reasons for this attitude. First of all, what possible physical interest could there be in problems for which existence, uniqueness or perhaps continuous dependence failed? Hadamard himself remarked "But it is remarkable, on the other hand, that a sure guide is found in physical interpretation: an analytic problem always being correctly set... when it is the translation of some mechanical or physical question." Another justification for the lack of interest in such problems is given by Courant [2]. "Unfortunately little mathematical progress has been