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Finite element Galerkin methods for differential equations, by Graeme Fairweather, Marcel Dekker, Lecture Notes in Pure and Applied Mathematics, vol. 34, New York, Basel, 1978, ix + 263 pp., $19.75.

Elliptic boundary value problems come up often in applications. These problems are usually solved numerically. A simple approach is to replace each derivative with a difference quotient–this is called the method of finite differences. Thus the solution of the differential equation is approximated by approximating the differential equation. Another approach is to approximate the solution of the differential equation directly. The Finite Element Method does this as follows: Consider the differential equation to correspond with a variational principle. This variational principle is solved approximately. The approximation is that a finite family of functions replaces the infinite set of functions that satisfy the boundary conditions and are sufficiently smooth for integration by parts. The creative step is to determine this finite family suitably. For concreteness, consider the domain of definition for the differential equation to be a nice region in the plane, a polygon perhaps. The region is divided up into pieces. Over each piece, suitable approximating functions are defined. An example is dividing a polygonal region into triangles, followed by piecewise linear interpolation over the triangles. The term "finite elements" refers either to the triangles or to the linear functions. (Which it is, depends on the author.) Of course, piecewise linear interpolation cannot be carried out, because the values of the solution of the differential equation are not known inside the region. But these values are determined by a least squares approximation to the solution, in the inner product corresponding to the variational principle. The Finite Element Method is also called the Rayleigh-Ritz-Galerkin Method, since these are particular cases of it.

Perhaps an example helps: The differential equation is Poisson's equation

$$-\Delta u = f. \tag{1}$$

The approximation is of the form

$$u(x, y) \simeq \sum_{i=1}^{n} A_i B_i(x, y) \tag{2}$$

where the $B_i(x, y)$ are the basis functions and the A_i are numbers to be determined by the Finite Element Method, as follows:

$$\sum_{i=1}^{n} A_i a(B_i, B_j) = (f, B_j), \qquad j = 1, \ldots, n, \tag{3}$$

where $a(\cdot, \cdot)$ is the "energy" inner product corresponding to (1) and $(\cdot, \cdot)$ is the usual L_2 inner product.

The history of the Finite Element Method is interesting. It has been used by engineers to get numerical answers to problems for some time now. During the past decade, mathematicians have gotten interested in it. Their main successes have been with linear problems, e.g., Laplace's equation, which is the Euler equation corresponding to the variational problem of minimizing the L_2 norm of the gradient, subject to certain boundary conditions. Mathematicians have had some success analyzing elliptic, hyperbolic, and parabolic partial differential equations and for two-point boundary value problems in ordinary differential equations. Some of these successes are chronicled in Fairweather's book. The book is for mathematicians–it is too theoretical to interest many engineers.

The best all around book on the Finite Element Method for mathematicians is *An analysis of the finite element method*, by G. Strang and G. Fix. Their book covers many aspects of the Finite Element Method, including some computational and practical considerations. Also, P. G. Ciarlet has just completed a book similar in emphasis to Fairweather's: *The finite element method for elliptic problems*, North-Holland Publ. Co., Amsterdam & New York, 1978.

The Finite Element Method is widely used and hence is important to a wide group of people. Most of the basic schemes were known to engineers prior to their being theoretically well-grounded by mathematicians. A useful contribution by mathematicians has been to invent new schemes for special problems where the standard schemes fail.

A recent development has been the use of computer graphics to display finite element results. This has considerable promise.

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