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Classical Banach spaces. I, by Joram Lindenstrauss and Lior Tzafriri, Ergebnisse der mathematik und ihrer Grenzgebiete, no. 92, Springer-Verlag, Berlin, Heidelberg, New York, 1977, xiii + 190 pp.

Although it was difficult to prove theorems about Banach spaces fifteen years ago, it was certainly easy to learn Banach space theory. Indeed, after going through Banach's classic [1] one needed only to fill in with sections from Day's book [2] and read a few papers. By 1970, the subject had developed tremendously, but the novice could still enter easily the mainstream of Banach space theory. Now, however, despite the appearance in recent years of a number of expository monographs and textbooks centered around the subject ([3], [4], [5], [6], [8], [9], [10], [11] and [12]), students and nonspecialists find it difficult to develop a broad understanding of Banach space theory. Indeed, the aforementioned books pick out relatively narrow parts of the subject (and, in some cases, present them well), but they do not give a broad perspective of Banach space theory as seen by leaders in the field.

In their Springer Lecture notes [7], Lindenstrauss and Tzafriri summarized "the main directions and current problems in Banach space theory". These notes apparently were well received, but their usefulness was restricted by the lack of proofs. Moreover, many of the problems mentioned in [7] were solved shortly after (or even before) [7] appeared in print, so [7] was already out of date by 1974. Consequently, Lindenstrauss and Tzafriri decided to write a three volume (now four volumes) expanded and updated version of [7] entitled *Classical Banach spaces*, broken down as follows: volume 1, *Sequence spaces*; volume 2, *Function spaces*; volume 3, *L_p and $C(K)$*; volume 4, *Local theory*.

Volume 1 begins, naturally enough, with a discussion of the main part of basis theory. Here you will find, for example, the classical results on the characterization of reflexivity in terms of properties of bases, the usual material on unconditional bases, and gliding hump procedures (included is the recently developed blocking technique as well as the classical gliding hump arguments which every student must know). Significant open problems involving these notions in general spaces are discussed, for example, on page 27 appears the most important (in the reviewer's opinion) unsolved problem in the theory of general Banach spaces–"Does every infinite-dimensional Banach space contain an unconditional basic sequence?"–and on the next page you find the Maurey-Rosenthal example of a weakly null normalized sequence which has no unconditionally basic subsequence.

In view of the fact that entire books have been written on general basis theory, it is worthy of note that Lindenstrauss and Tzafriri have managed to present most of the important part of this material in 47 pages.

The well-developed structure theory for l_p and c_0 is contained in Chapter 2. The spaces which have a unique unconditional basis are classified as being l_1,

l_2, or c_0. The classification of complemented subspaces l_p and c_0, and even that of $l_p + l_r$ (which requires the development of spectral theory for strictly singular operators) is included, as is a proof of the result that a subspace of l_p which has a finite-dimensional decomposition is an l_p sum of finite-dimensional spaces. This section also discusses the relationship of l_p and c_0 to general Banach spaces, in particular, the fundamental characterizations of Banach spaces which contain l_1 are presented. Finally, the many positive results are balanced by an example of a Banach space which contains no copy of any l_p or c_0, and of course a method for constructing subspaces of l_p $(p > 2)$ which fail the approximation property is given.

The fact that every unconditional basic sequence is a block basis of a symmetric one points out the difficulty in dealing with general symmetric bases; nevertheless, there is a nice general theory which is presented well in Chapter 3. This theory is specialized in Chapter 4 to Orlicz and Lorentz sequence spaces. For example, the subspaces of an Orlicz sequence space l_M which have a symmetric basis are classified as those Orlicz spaces l_N for which N is related to M in a certain way; the satisfactory duality of Orlicz spaces is explored (e.g., l_p^* is a subspace of l_M^* iff l_p is a subspace of l_M); and the difficult known results concerning complementation in Orlicz spaces are proved.

The authors have certainly made a good beginning toward achieving their goal of presenting "the main results and current research directions in the geometry of Banach spaces, with an emphasis on the study of the structure of the classical Banach spaces, that is $C(K)$ and $L_p(\mu)$ and related spaces". Since the material in volume 1 is pretty well understood by the specialist, its main value for the expert is that the book provides a convenient reference, with good proofs which do not have unnecessary detail or computation, of the most important results on sequence spaces.

For the student, this book is indispensible. A word of warning to the student and casual reader is, however, advisable. The emphasis in the book is on recent results and new ideas. Standard (i.e., known by the authors for some time) techniques are considered easy; results which have been around for a while are called simple. The student who fills in the gaps has done a lot, and his work begins on page 1: in order to prove that a basis $\{e_i\}_{i=1}^{\infty}$ for a Banach space has continuous coordinate functionals, the new norm

$$\left|\left|\left| \sum_{i=1}^{\infty} a_i e_i \right|\right|\right| = \sup_n \left\| \sum_{i=1}^{n} a_i e_i \right\|$$

is defined on X, and one must verify that $(X, |||\cdot|||)$ is complete. Banach's proof of this is perhaps a "simple argument", but my experience suggests that students must work hard to check the completeness of $|||\cdot|||$.

This fine book will surely be the Bible on Banach spaces for many years, since the authors are undoubtedly correct in their feeling "that most of the topics discussed here have reached a relatively final form, and their presentation will not be radically affected by the solution of the open problems".

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*Induced representations and Banach *-algebraic bundles*, by J. M. G. Fell, with an appendix due to A. Douady and L. Dal Soglio-Hérault, Lecture Notes in Math., vol. 582, Springer-Verlag, Berlin, Heidelberg, New York, 1977, 349 pp., $13.70.

Let G be a finite group whose representation theory one wishes to understand. A natural strategy is to consider a subgroup H of G, whose representation theory is presumably simpler, and try to use representations of H to construct representations of G. A method for making such a construction, that of induced representations, was introduced by Frobenius in 1898 [**8**], and has played a central role in representation theory ever since.

If R is a ring whose representation theory (= module theory) one wishes to understand, then one can use a similar strategy, by considering a subring, S, of R and trying to use S-modules to construct R-modules. General constructions for doing this have become well known as "change-of-ring" operations. Specifically, if M is a left S-module, and if one views R as a right S-module, then one can form $R \otimes_S M$, which is a left R-module. It was not until 1955 that it was pointed out, by D. G. Higman [**13**], that by using the group algebra of a group, Frobenius' definition of induced representations can be viewed as a special case of this change of ring operation. (Probably the reason this was not noticed earlier is that tensor products in this noncommutative setting had not been clearly formulated until a short time before.) Of course in the group case one goes on to exploit the richer information