for a beginner. This book is an important and valuable addition to a growing secondary literature, which is perhaps best absorbed by wandering back and forth from one book to another and thence to the primary sources as quickly as possible.

We wish to add our thanks and compliments to J. N. Crossley for translating this work into (very readable) English.

REFERENCES

1. J. Bridge, *A simplification of the Bachmann method for generating large countable ordinals*, J. Symbolic Logic **40** (1975), 171–185.

2. W. Buchholz, *Eine Erweiterung der Schnitteliminations methode*, Habilitationschrift, Ludwig-Maximilians-Universität, München, 1977.

3. S. Feferman, *Formal theories for transfinite iterations of generalized inductive definitions and some subsystems of analysis*, Intuitionism and Proof Theory, North-Holland, Amsterdam, 1970, pp. 303–325.

4. ———, Review of [12], Bull. Amer. Math. Soc. **83** (1977), 351–361. (Note: [F2] of the references of [4] has now appeared in the *Handbook of mathematical logic* (J. Barwise, ed.), North-Holland, Amsterdam, 1970, pp. 913–971.)

5. H. Friedman, *Iterated inductive definitions and* Σ^1_2-AC, Intuitionism and Proof Theory, North-Holland, Amsterdam, 1970, pp. 435–442.

6. J.-Y. Girard, *Three-valued logic and cut-elimination: the actual meaning of Takeuti's conjecture*, Dissertationes Math. **136** (1976).

7. W. Pohlers, *Beweistheorie der iterierten Induktiven Definitionen*, Habilitationschrift, Ludwig-Maximilians-Universität, München, 1977.

8. K. Schütte, *Beweistheorie*, Springer-Verlag, Berlin, 1960.

9. W. Sieg, *Trees in metamathematics: theories of inductive definitions and subsystems of analysis*, Dissertation, Stanford Univ. 1977.

10. S. Stenlund, *Combinators, λ-terms and proof theory*, Reidel, Dordrecht, 1972.

11. W. W. Tait, *Infinitely long terms of transfinite type*, Formal Systems and Recursive Functions (Crossley and Dummett, eds.), North-Holland, Amsterdam, 1965, pp. 465–475.

12. G. Takeuti, *Proof theory*, North-Holland, Amsterdam, 1975.

13. A. S. Troelstra (Editor), *Metamathematical investigation of intuitionistic arithmetic and analysis*, Lecture Notes in Math., vol. 344, Springer-Verlag, Berlin, 1973.

SOLOMON FEFERMAN

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 1, Number 1, January 1979

0002-9904/79/0000-0010/$01.75

Linear estimation and stochastic control, by M. H. A. Davis, Wiley, New York, 1977, xii + 224 pp., $14.95.

In this monograph the author gives a highly readable introduction to two topics, namely the Kalman filter and the stochastic linear regulator problem. These two topics have been called the "bread and butter" of modern stochastic systems theory. They have the fortunate feature that the mathematical techniques used are elegant, while at the same time the results have been quite widely used in engineering and other applications. Rather modest background is needed to read the book. The equivalent of introductory real-analysis, probability, and some familiarity with elementary linear systems theory should suffice.

The linear estimation problem is as follows. Given random variables X, Y_s for $s \in S$, all of zero mean and finite variance, find the approximation $\hat{X}$ to X

in the Hilbert space spanned by $\{Y_s\}$ which is best in mean square. One may think of X as being unobservable, and $\{Y_s\}$ as representing the available data. Often one has not a single random variable X, but rather a stochastic process X_t. Moreover, new data is arriving continually; the observations at time t are $\{Y_s\}$ for $s \leq t$, with now the index set $S = S_t$ the interval $[0, t]$. What is needed is a computationally tractable scheme for updating the best linear estimate $\hat{X}_t$. In 1960 Kalman introduced such a technique, which was then developed further by Kalman-Bucy and others. It now goes by the name "Kalman filter".

In the Kalman filter model X_t, Y_t obey linear differential equations

$$dX_t = A_t X_t \, dt + C_t \, dV_t, \tag{1}$$

$$dY_t = H_t X_t \, dt + G_t \, dW_t, \tag{2}$$

where A_t, C_t, H_t, G_t are nonrandom, and V_t, W_t are certain stochastic processes with orthogonal increments. [The stochastic processes X_t, Y_t, V_t, W_t are allowed to be vector-valued; then $A_t, C_t, \ldots$ are matrices.] The main Kalman filter results are: (a) $\hat{X}_t$ also obeys a certain linear differential equation, driven by the observation process Y_t; (b) the covariance matrix of the error $X_t - \hat{X}_t$ does not depend on the observations. In fact, these covariance matrices can be precomputed by solving a system of ordinary differential equations of Riccati type. A key to the method is to introduce the "innovations process"

$$V_t = Y_t - \int_0^t H_s \hat{X}_s \, ds.$$

This process has orthogonal increments; and $\{V_s\}$ for $s \leq t$ generate the same Hilbert space as $\{Y_s\}$ for $s \leq t$.

In the stochastic linear regulator problem, equation (1) is replaced by

$$dX_t = (A_t X_t + B_t U_t)dt + C_t \, dV_t, \tag{3}$$

where U_t represents a control used at time t. The control may depend on past observations $\{Y_s\}$ for $s \leq t$. The control process U_t is to be chosen to minimize a quadratic of the form

$$E\left\{\int_0^T (X_t' Q_t X_t + U_t' R_t U_t)dt + X_T' F X_T\right\}. \tag{4}$$

A "separation principle" reduces this problem to two others. One is a Kalman filter to find $\hat{X}_t$, the other is a deterministic linear regulator problem in which the estimate $\hat{X}_t$ is regarded as the true state of the system. A solution to the deterministic linear regulator problem can be found by solving a system of ordinary differential equations of Riccati type.

In a brief final chapter, a taste of recent developments in nonlinear filtering and stochastic control is given. For an excellent, comprehensive treatment of nonlinear filtering see R. S. Liptser and A. N. Shiryayev, *Statistics of random processes*, vols. I, II; English transl., Springer-Verlag, Berlin and New York, 1977.

WENDELL H. FLEMING