ting zeros of polynomials and includes an elegant axiomatic treatment of computer arithmetic.

Since the notes were intended for elementary courses, they presuppose little mathematical background on the part of the reader. A thorough familiarity with calculus and elementary linear algebra is all that is required for most of the work. However, this does not greatly limit its scope, since most numerical algorithms can be derived from rather elementary considerations, even though a complete analysis may require a great deal of mathematical apparatus. Moreover, the elementary approach has enabled the author to segregate his topics into essentially independent essays. This is no doubt more the result of the lecture note format than of design, but I find the style quite congenial to the eclectic nature of numerical analysis.

As might be expected from the circumstances of their publication, the notes are uneven, with some parts having more polish than others. More seriously, much of the work is out of date. No mention is made of the use of finite elements to solve partial differential equations; nor is the QR algorithm mentioned in the sections on algebraic eigenvalue problems. I found myself wishing that the editor had appended annotated references to more recent works. Not only would this have increased the value of the notes, but it also would have reduced the chances of the casual reader's being misled about the current state of the art.

However, the virtues of the work far outweigh its defects. It is unfortunate that it is available only in German; for it deserves to be more widely read. Rutishauser's audience is not only the student, but the instructor teaching a numerical analysis course for the first time, and especially the mathematician who wants to find out what this important branch of applied mathematics is all about.

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Lie groups and compact groups, by John F. Price, London Mathematical Society Lecture Note Series, no. 25, Cambridge Univ. Press, Cambridge, London, New York, Melbourne, 1977, ix + 177 pp., $8.95.

There are few truly successful classification theorems in mathematics–that is, theorems which describe all examples of an apparently large class of objects in a relatively simple and constructive way. One of the best such theorems classifies compact connected Lie groups. As is usual in the subject, I shall use the word "simple" to mean what is also called "almost simple": G is simple if it has a finite center C such that G/C has no nontrivial closed normal subgroups. Now let G be any compact, connected Lie group, let Z be the connected component of the identity in the center of G, and let $H = [G, G]$, the closure of the commutator subgroup of G. The classification theorem says that Z is (isomorphic to) a torus and that $G \cong Z \times H/F_0$, where F is a finite central subgroup. Moreover, $H \cong G_1 \times \cdots \times G_n/F$, where $G_1, \ldots, G_n$ are simply connected simple Lie groups (uniquely determined up to order by H) and F is a finite central subgroup. Finally, each factor G is one of a specific list of groups, composed of certain classical groups (or their simply connected covers) and the five exceptional groups G_2, F_4, E_6, E_7, and E_8. This statement, the classification of the simply connected compact groups, is one of the most beautiful results in mathematics.

The history of the theorem begins with Lie, who pointed out that the Lie algebras of the classical groups **C** were (with two exceptions) simple complex Lie algebras. Killing, in a series of papers [4], then showed that there were at most six other simple complex Lie algebras, which he named G_2, F_4, E_4, E_6, E_7, and E_8. In his 1894 thesis [1], Cartan corrected a number of mistakes in Killing's reasoning, showed that E_4 and F_4 were isomorphic, and constructed the five exceptional algebras. Later (in [2]) Cartan showed that the complex simple Lie algebras were in 1-1 correspondence with the local isomorphism classes of compact simple Lie groups. After Weyl showed (in [5]) that the universal covering group of a compact simple Lie group is itself compact, Cartan was able to give the characterization of compact Lie groups (though in a form slightly different from the one given above: see [3]).

The classification theorem is important for other than aesthetic reasons, too. Much of the material which goes into proving it, such as Cartan subalgebras, roots and weights, and the analysis of real forms, underlies further work in Lie groups, including Harish-Chandra's massive and splendid theory. Furthermore, some theorems about compact Lie groups have been proved by a case-by-case analysis, using the classification. As Lie groups assert their central position in more and more areas of mathematics, these results become more and more familiar to the working mathematician.

Nonetheless, they are not part of the standard curriculum for graduate students (as measured, for example, by what is fair game on Ph.D. qualifying exams). The reason is not hard to find. Proving the results involves knowledge of differential geometry, the various results which describe the properties of the functor taking a Lie group G to its algebra $\mathfrak{G}$, and a large dose of Lie algebra theory. It helps to know something of the representation theory of compact groups as well. To get through all this material requires a fair amount of sophistication and time.

It also requires a hefty book. The book under review, whose stated objective is to provide a quick self-contained introduction to the general theory of Lie groups and to give the structure of compact connected groups and Lie groups in terms of certain distinguished simple Lie groups, is certainly not hefty. It begins at the beginning, with the definition of a differentiable manifold, and concludes with the structure theorem for connected compact Lie groups and a related theorem for general connected compact groups. The compression is necessarily achieved at a price. In this case, the price is completeness; many important theorems are quoted and not proved. There is nothing about the classification of simply connected simple compact Lie groups except a statement of the result: and Weyl's theorem on covering groups (cited earlier) and Cartan's criterion for semisimplicity of Lie algebras (to give just two examples) are used without proof.

To my mind, the result comes close to throwing out the baby and keeping the bathwater. The proof that, for instance, every subalgebra of a Lie algebra corresponds to a Lie subgroup, strikes me as one of those necessary but dull routines which are found in the elements of almost every mathematical subject. The classification of simple Lie algebras, on the other hand, is a stunning example of how beautiful linear algebra can be. The inclusion of the classification theorem at the end is a delicacy for the medicine the student has been forced to swallow; when so much of the proof is missing, however, the student is unable to recognize the treat for what it is.

There seem to be only a few misprints; e.g., the reference to A.3.9 on p. 146 should be A.1.6, and "notamment" is misspelt on p. 152. None of them should cause trouble. Each of the six chapters ends with historical notes and some exercises; many of the latter extend remarks and fill gaps in the text

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Mathematical theory of economic dynamics and equilibria, by V. L. Makarov and A. M. Rubinov, Springer-Verlag, New York, Heidelberg, Berlin, 1977, xv + 252 pp. $32.10.

In 1932 von Neumann presented a paper at a mathematics seminar at Princeton which was published six years later at the request of K. Menger [5] and translated into English in 1945 [6]. The ideas which form the main theme of the book under review, especially the key concepts of "economic dynamics" and its relation to "equilibria", appeared here in explicit mathematical form, probably for the first time. The von Neumann paper is one of the two "primary sources" for this book (the other will be mentioned later), and as such seems a convenient starting point for this review.

Here is a paraphrase of what von Neumann did. He considered an economic world in which there are n goods and a system of production, a *technology*, which can be described in the following simple manner. A *process* consists of a pair (a, b) of nonnegative n-vectors $(a_1, \ldots, a_n)$, $(b_1, \ldots, b_n)$ where the entries a_i and b_i represent the amounts of the ith good. The physical interpretation is that if the vector a is available in an *input* at some time t then the vector b can be obtained from it as an *output*, becoming available at time $t + 1$. Processes are assumed to be *positively homogeneous* so that if (a, b) is a process so also is $(\xi a, \xi b)$ for any $\xi \geqslant 0$. Further if (a, b) and (a', b') are processes it is assumed that they may be operated simultaneously so that $(a + a', b + b')$ is also a process. A technology T is simply a set of