these things should fit into a general framework are challenges that should be given serious consideration by any student of transcendental number theory.

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<div align="right">CARLOS MORENO</div>

Combinatorial optimization: networks and matroids, by Eugene L. Lawler, Holt, Rinehart and Winston, New York, 1976, x + 374 pp.

This is a well-written introduction to an attractive area of modern mathematics. It is highly recommended.

Some problems in this area are:

1. Find the shortest path through a finite network.

2. Find the kth shortest path through a finite network.

3. Find the path of shortest length through all points of a finite network ("the travelling salesman" problem or technically a Hamiltonian circuit.)

4. How does one process m items on n machines?

5. How does one calculate 2^n with a minimum number of multiplications?

6. How does one compute a polynomial in many variables with a minimum number of multiplications?

7. How does one find m defective coins among n coins?

The fourth, fifth, sixth, and seventh problems are not treated in this book. The fourth problem is very important in many industrial applications and in operating a computer installation. Nabeshima has written a book in Japanese on this problem, which he is translating into English. Many other mathematicians have worked on this problem. Branch and bound techniques have been used by many. The fifth problem has no applications that the reviewer knows of. It is like many problems in number theory, simply stated and intractable. The sixth problem has many applications in a number of algorithms. In this case of polynomials of one variable, the problem is solved. Ostrowski treated the case of polynomials up to degree four, and the general case was treated by Pan. They showed that the well-known technique of Horner was best.

In problem 7 the case $m = 1$ is a well-known puzzle which may be solved using many methods. The case $m = 2$ was treated in [1]. The case of general m is part of a mathematical theory of experimentation which does not yet exist.

Since these are finite problems and we have a digital computer at our

disposal, it might be thought that they could readily be solved by enumeration. This is not the case. A convenient unit for combinatorial problems is 10!. This number is 3,628,800. 20! is easily seen to be more than 10^{10} times as large. Consequently, whatever time is required to examine 10! cases, 20! cases will require 10^{10} times as much. Yet, simply stated combinatorial problems lead to 100! or 1000! cases and even a larger number of cases.

Another convenient unit is that a year has approximately $\pi \times 10^7$ seconds. We see then, that even at microsecond speed enumeration is not a feasible procedure. Mathematical analysis is required.

What is so attractive about these problems is that they require a blend of analysis, algebra, topology, computer science and often, a knowledge of where the problem arose. This origin often provides valuable clues to the nature of the solution. At very least, it furnishes a very useful first approximation. This is true in many parts of analysis. Thus, they force the mathematician to learn many branches of mathematics. If he hopes for success, he cannot be a narrow specialist.

A knowledge of graph theory is indispensable. (An excellent book on this subject is [5].) The book under review contains a chapter on the shortest path problem. This is one of the basic problems of modern mathematics. As the author shows, many problems can be solved directly in terms of this problem. Many other problems use this problem as part of the solution. For example, Dreyfus in his work on the Steiner problem uses the shortest path problem.

A simple dynamic programming argument yields a basic nonlinear equation. Thus, the original combinatorial question has been converted into an analytic one. This equation can be treated in various ways by successive approximations which yield upper and lower bounds. Many of the approximations have simple interpretations as approximate policies.

The author also gives many other ways of treating this problem in this chapter and a later chapter.

This problem also plays a major role in the calculus of variations (see [3]); it can also be used to treat a number of puzzles (see [2]); and, finally, it can be made a fundamental part of decision making by a digital computer (see [4]).

The later chapters are more technical and contain a great deal of work original with the author.

One of the great merits of the book is that it shows how much research remains to be done. First of all, there is no such thing as a final solution of any of these problems. The algorithms that are used depend upon mathematical analysis as well as the development of computers. As parallel computers become available, there will be many new algorithms. Secondly, we face the problem of deriving feasible procedures where no mathematical analysis exists. Thirdly, all of these problems can be extended to the case where stochastic effects are present. We can expect an interesting blend of classical topology and stochastic processes. Fourthly, all of these problems can be

extended to the case where there is learning. It is important to consider the fuzzy versions of these problems. The word "fuzzy" is used in the sense of Lotfi Zadeh.

Finally, a historical note . Problems of this type were considered by many mathematicians: Euler, Hamilton and Steiner, to name a few. But the first systematic study of these problems was carried out at the RAND Corporation during the years after 1948 under the inspiration, and often participation, of von Neumann. Major names were: George Dantzig, Stuart Dreyfus, Lester Ford and Ray Fulkerson. Many other mathematicians worked on these problems. They are closely connected with the theory of games, linear and nonlinear programming, as well as integer programming.

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Doubly stochastic Poisson processes, by Jan Grandell, Lecture Notes in Mathematics, vol. 529, Springer-Verlag, Berlin, Heidelberg, New York, 1976, x + 234 pp., $10.30.

A point process on a space T is a random distribution of points throughout T. Its values are atomic measures on the space with the atoms having weights $1, 2, 3, \ldots$ (corresponding to $1, 2, 3, \ldots$ points at the atoms). An ordinary stochastic process refers to a random entity whose possible values are functions. In contrast, a point process refers to a random entity whose possible values are counting measures. Inherent and central to the notion of such a process is the idea of whether the points tend to be abundant and closely packed or sparse and widely separated, i.e. their intensity. To formalize this idea, suppose I is a measurable subset of the space. Suppose $N(I)$ denotes the number of points that are in I for a realization of the process. Then the expected or average value of $N(I)$, $E\{N(I)\} = \mu(I)$, is called the intensity measure of the process. In the case that the space is the real line, and the measure μ is absolutely continuous, its derivative is called the intensity function of the process. A point process is called Poisson with intensity measure μ if (i) for measurable I,

$$\text{(1)} \qquad \text{Prob}\{N(I) = n\} = \mu(I)^n \exp\{-\mu(I)\}/n!,$$