group over a finite field is an excellent example of a Sylow subgroup. It is this sort of blend of specific and general which seems to make the best mathematics. An example without a theory to understand it is just as dry and uninteresting as an abstract theorem with no illustrative example to bring it to life.

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Homogeneous Banach algebras, by Hwai-Chiuan Wang, Lecture Notes in Pure and Applied Mathematics, Volume 29, Marcel Dekker, Inc., New York and Basel, 1977, vii + 204 pp., $19.75.

Since the appearance of Gelfand's work on commutative Banach algebras [6] the ideas of that subject have come to be an integral part of many areas of analysis. Nowhere is this more so than in harmonic analysis, where a significant portion of the research of the past thirty years rests upon ideas and questions inspired by Gelfand's work.

One of the first fruits of Gelfand's theory was a re-examination of the foundations of harmonic analysis on locally compact Abelian (LCA) groups G; the fundamental links between the algebra $L^1(G)$, its representations and the characters of G, Bochner's theorem on positive-definite functions, the inversion theorem for Fourier transforms, and so on, were discovered, or looked at afresh. (See the first two chapters of Rudin's well-known book [13] for an explanation of these matters.)

More significant and exciting was the fact that questions of an algebraic kind began to be asked in the domain of harmonic analysis. For instance, what do the closed ideals of the convolution algebra $L^1(G)$ look like? What are the closed subalgebras of $L^1(G)$? What is the structure of the maximal ideal space of the convolution algebra $M(G)$, of all regular Borel measures on G? What are the functions that "operate" on the space of Fourier transforms of this or that algebra? Around questions such as these, rich subcultures of harmonic analysis have grown up, and continue to flourish today.

It was natural that, as questions of the kind indicated above were being asked about $L^1(G)$ and $M(G)$, the same, or related, questions should be asked about various other algebras of functions or measures. (So, for instance, the subject of function algebras, with rich links to both harmonic analysis and function theory, grew up.)

In this spirit, Reiter introduced in [8] the notion of a Segal algebra. By definition, a *Segal algebra* on G is a Banach subalgebra of $L^1(G)$ such that

(i) A is dense in $L^1(G)$;

(ii) $\|f\|_1 \leq \|f\|_A$ for all f in A;

(iii) A is translation-invariant;

(iv) the operation of translation τ_a: $\tau_a f(x) = f(x - a)$ is, for every a, an isometry on A;

(v) the mapping $a \mapsto \tau_a f$ is continuous from G into A, for each f in A.

For example, $L^1 \cap C_0(G)$, $L^1 \cap L^p(G)$, $A^p(G) = \{f \in L^1(G): \hat{f} \in L^p\}$, $1 \leq p < \infty$ ($\hat{f}$ denoting the Fourier transform of f) are, with their natural norms, Segal algebras. Another interesting example is the space $W(\mathbf{R})$ introduced by Wiener [15].

$$W(\mathbf{R}) = \left\{ f \in L^1 \cap C(\mathbf{R}): \sup_x \sum_{k=-\infty}^{\infty} \max_{t \in [0,2\pi]} |f(x + t + 2k\pi)| < \infty \right\}.$$

What Reiter showed was that in certain respects, Segal algebras are like the algebra $L^1(G)$ itself. For instance, a Segal algebra has the same maximal ideal space as $L^1(G)$; the theory of its closed ideals is the "same" as that of $L^1(G)$. The book [9] and lecture notes [10] of Reiter should be consulted for a detailed treatment of Segal algebras; the lecture notes include, incidentally, extensions of the theory to noncommutative groups. A lengthy historical account of the subject is included in Burnham's article [1].

The homogeneous Banach algebras, which are the subject of the present book, are those subalgebras of $L^1(G)$ which have properties (ii)–(v) above. Because a homogeneous Banach algebra is not necessarily dense in L^1, one cannot expect as close an association between their theory and that of L^1. On the other hand, the class of homogeneous Banach algebras includes very many of the well-known and less-well-known spaces of functions, as Wang amply demonstrates. The author covers many by-now-classical questions in the general context of homogeneous algebras: factorization and nonfactorization, closed subalgebras, maximal ideal space, automorphisms, closed ideals. In various places there are particularizations to and contrasts with the Segal, and other algebras. For instance, examples are included of a Segal algebra which is not stable under multiplication by characters; and another which is not stable under the natural involution. These examples answer questions raised by Reiter (loc. cit.).

An important tool in the theory is that of factorization. Cohen [3] proved that if A is a Banach algebra with bounded approximate identity, then there is a number $K > 0$ such that, whenever $a \in A$, and $\varepsilon > 0$, there exist b, $c \in A$ such that $\|a - b\| < \varepsilon$, $\|c\| \leq K$, and $a = bc$. Cohen's theorem was subsequently extended to the case of modules over a Banach algebra (Curtis-Figà-Talamanca [4], Hewitt [6]). The latter result showed, for instance, that $L^1 * C_0(G) = C_0(G)$. Besides using the module factorization theorem extensively, Wang devotes a chapter of his book to the question of factorization, in various senses, in Segal algebras. He gives numerous examples of Segal algebras where factorization (in the sense that each element should be expressible as $g * h$, without restriction on the norms of the factors) fails. The arguments are straightforward, and most rest on the observation that if $1 < p < \infty$, and n is large enough, then the pointwise product $f_1 \cdots f_n$ of n elements of L^p is integrable. It is apparently still an open question whether a Segal algebra can be such that $A^2 = A$ without having a bounded approximate identity (and hence being all of $L^1(G)$).

A general observation about this book is that very many of the results presented are in the nature of fairly straightforward corollaries of deeper results quoted without proof (for instance, of the factorization theorems mentioned above, of the Kahane-Rider results on closed subalgebras of $L^1(G)$ [7], [11], of Cohen's analysis of the homomorphisms of $M(G)$ [2], and so on). My view is that a good book on a research topic ought to contain a number of really challenging results for the reader to come to grips with, and to mull over later. If this is accepted as a valid criterion, then Wang's book will leave a lot of people unsatisfied.

In matters of presentation, this book leaves a great deal to be desired. It may seem harsh to criticise numerous misuses of language in a book by an author for whom English is a second language. However, it is not only the author who should bear the brunt of such criticism; it is also the persons responsible for the production of his book. When one finds sentence after sentence that does not, by any stretch of the imagination, read decently, and a confusion of similar-sounding but different words (e.g. "conversion" instead of "converse") then the conclusion has to be that the editorial staff neglected their job. On the other side, neither is it unreasonable to express the regret that the author did not trouble to have his typescript read over by a native-speaking colleague.

This book is about a nice circle of ideas with some interesting, still-unsolved problems. The subject does not currently occupy the centre stage of research in harmonic analysis; however, it has some good things to offer, and it is a pity that the present treatment of it was not just that much better.

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Hausdorff compactifications, edited by Richard E. Chandler, Marcel Dekker, Inc., New York and Basel, 1976, vii + 146 pp., $16.50.

Work on compactifications began in 1924 with Tietze, Alexandroff and Urysohn. In 1930, Tychonoff characterized completely regular (Hausdorff)