penetrate the jungle of technical details and become fascinated by the kaleidoscopic picture which I have tried to unfold here of the history of the first and oldest natural science".

One can only hope that a future historian will be able to accomplish as much when the astronomy of the twentieth century has itself been reduced to a few odd books and some handfuls of fragments.

N. SWERDLOW

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Examples of groups, by Michael Weinstein, Polygonal Publishing House, Passaic, N.J., 1977, 307 pp.

"Why study examples?" asks the author as he opens his preface to this curious volume. Why, indeed. This is a question which I think many of today's graduate students and more than a few of their instructors could ponder profitably. The author gives us three reasons:

(1) to motivate new theorems,
(2) to illustrate and clarify old theorems,
(3) to obtain counterexamples.

While all of this is well put and certainly true, it seems to me that the main reason for studying examples is simply that we can't do without them. What, after all, is a theorem if it is not a simultaneous assertion about some properties of a large class of examples? What better way to understand what a theorem says than to apply it to some concrete examples? Everyone appreciates the power and desirability of generalization. Studying examples is just the reverse process of going from the general to the specific. Mathematics without examples would become the uninteresting exercise in formal deduction which it is sometimes mistaken for.

Unfortunately, the study of examples is seldom given the status which it deserves, particularly in some modern texts, and the present book is an admirable attempt to rectify this situation, as it pertains to the theory of discrete groups. How well does it succeed?

The author presents us with a rather long list of specific discrete groups; finite and infinite, abelian and nonabelian. In each case, a number of properties are obtained. For example, turning (at random) to p. 194, we see: "Result 5.11.5. G is not an M_1 group. Result 5.11.8. G is Hopfian." There is also a section of comments ("notes") following each example (e.g. "G shows that the class of cohopfian groups is not quotient closed") and a number of exercises. The first example appears on p. 101 and is preceded by an entire section devoted to some abstract construction techniques (e.g. direct, central, semidirect, and wreathed products) and some elementary facts about free groups and matrix groups. Additional elementary results appear in a series of ten appendices. All arguments are given in a very careful and complete manner, but the price we pay for this is a rather pedantic and heavy style:

"If k is a natural number such that $2 < k$, then 2 and k are distinct divisors of $k!$, and hence $2k \leqslant k!$. Also $2 < k$ implies $1 \leqslant k$ so $1 + k \leqslant 2k$. Thus $1 + k \leqslant k!$ for all k such that $2 < k$."

In addition to this formal and somewhat stilted style ("If A is a 2 by 2 matrix, then $\det(A)$ is the determinant of A if and only if $\det(A) = A_{11}A_{22} - A_{12}A_{21}$") the exposition has an unpleasantly formal flavor. Almost all discussion of the individual examples relates to whether general property X is or is not satisfied in the given group. We are never told that such and such a group is interesting because it occurs in real life in a certain way.

Indeed, Example 4.4 is entitled "The Quaternion Group" but the connection with the real quaternions is relegated to Exercise 8. Even worse, Example 4.6 is entitled "The Tetrahedral Group"; but "Result 4.6.1" through "Result 4.6.11", Exercises 1 through 12, and Notes 1 through 6 all fail to reveal that this is the group of rigid motions of the tetrahedron! A similar problem exists with the octahedral and icosahedral groups. Example 5.5, "The p-adic Integers" is defined as $\mathrm{Hom}(Z(p^\infty), Z(p^\infty))$. In Note 2 we are told without proof that this group can be represented as the inverse limit of the finite cyclic p-groups. To the author's credit, he does state that this is "one of the most important properties . . . ".

Another problem with the exposition is the lack of any discussion about the generality of the examples presented. How representative are they? Reading through Chapter 4 ("finite groups") the naive reader might conclude that the set of groups $\{\mathrm{PSL}(2, q) | q = 2, 3, 5, 7, 9, 11\}$ is somehow representative of finite groups. What happened to $\mathrm{PSL}(n, q)$, $n \geqslant 3$, not to mention the symplectic, orthogonal, and unitary groups, the other groups of Lie type, or the sporadic groups? Other than a vague reference to the Mathieu groups and a mysterious mention of two nonisomorphic simple groups, both of cardinality 20160 (what is so mysterious about A_8 or PSL (3, 4)?) no hint is given that the surface has not been scratched. In the infinite groups section, there is no mention of various well-known examples, such as the Coxeter groups (this omission applies to the finite case as well), the Golod-Shafarevitch p-groups, the Higman universal finitely presented group, and so forth. All of this is not to say that the author should have included all these examples, merely that he should have informed the beginning student, for whom this book is clearly intended, that the given examples are of limited generality.

On a positive note, it is obvious that a lot of work has gone into the book. The arguments are painstakingly presented, there are lots of exercises, including a section at the back called "Hints to some of the exercises", and no less than five separate indexes. The book could certainly function effectively as a supplementary text in an advanced undergraduate algebra course.

My main problem with this book is not in its execution, but rather in its conception. To go back to the beginning, the reason why we study examples is that they are inextricably bound to the theory–we literally can't do without them. Therefore, a book about examples without theory is just as bad as the more frequently encountered book about theory with no examples. Examples cannot be coherently tied together without theory. Who cares if property A does or does not imply property B? For that matter, why are we looking at property A in the first place? What is needed are books which skillfully blend theory and examples. For instance, Sylow's theorem is stated in Appendix 10, p. 285. On pp. 71–79, we find a rather lengthy discussion of the unitriangular group, but as far as I can see no mention of the fact that the unitriangular group over a finite field is an excellent example of a Sylow subgroup. It is this sort of blend of specific and general which seems to make the best mathematics. An example without a theory to understand it is just as dry and uninteresting as an abstract theorem with no illustrative example to bring it to life.

DAVID M. GOLDSCHMIDT

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Homogeneous Banach algebras, by Hwai-Chiuan Wang, Lecture Notes in Pure and Applied Mathematics, Volume 29, Marcel Dekker, Inc., New York and Basel, 1977, vii + 204 pp., $19.75.

Since the appearance of Gelfand's work on commutative Banach algebras [6] the ideas of that subject have come to be an integral part of many areas of analysis. Nowhere is this more so than in harmonic analysis, where a significant portion of the research of the past thirty years rests upon ideas and questions inspired by Gelfand's work.

One of the first fruits of Gelfand's theory was a re-examination of the foundations of harmonic analysis on locally compact Abelian (LCA) groups G; the fundamental links between the algebra $L^1(G)$, its representations and the characters of G, Bochner's theorem on positive-definite functions, the inversion theorem for Fourier transforms, and so on, were discovered, or looked at afresh. (See the first two chapters of Rudin's well-known book [13] for an explanation of these matters.)

More significant and exciting was the fact that questions of an algebraic kind began to be asked in the domain of harmonic analysis. For instance, what do the closed ideals of the convolution algebra $L^1(G)$ look like? What are the closed subalgebras of $L^1(G)$? What is the structure of the maximal ideal space of the convolution algebra $M(G)$, of all regular Borel measures on G? What are the functions that "operate" on the space of Fourier transforms of this or that algebra? Around questions such as these, rich subcultures of harmonic analysis have grown up, and continue to flourish today.

It was natural that, as questions of the kind indicated above were being asked about $L^1(G)$ and $M(G)$, the same, or related, questions should be asked about various other algebras of functions or measures. (So, for instance, the subject of function algebras, with rich links to both harmonic analysis and function theory, grew up.)

In this spirit, Reiter introduced in [8] the notion of a Segal algebra. By definition, a *Segal algebra* on G is a Banach subalgebra of $L^1(G)$ such that

(i) A is dense in $L^1(G)$;

(ii) $\|f\|_1 \leq \|f\|_A$ for all f in A;

(iii) A is translation-invariant;

(iv) the operation of translation τ_a: $\tau_a f(x) = f(x - a)$ is, for every a, an isometry on A;

(v) the mapping $a \mapsto \tau_a f$ is continuous from G into A, for each f in A.

For example, $L^1 \cap C_0(G)$, $L^1 \cap L^p(G)$, $A^p(G) = \{f \in L^1(G): \hat{f} \in L^p\}$, $1 \leq p < \infty$ ($\hat{f}$ denoting the Fourier transform of f) are, with their natural