INFINITE DIMENSIONAL COMPACTA CONTAINING
NO n-DIMENSIONAL ($n \geqslant 1$) SUBSETS

BY JOHN J. WALSH

A compactum (compact metric space) is said to be *strongly infinite dimensional* provided there is a denumerable family $\{(A_k, B_k) | k = 1, 2, \ldots \}$ of pairs of disjoint closed subsets with the property that if, for each k, S_k is a closed subset which separates A_k and B_k, then $\bigcap \{S_k : k = 1, 2, \ldots \} \neq \varnothing$. The Hilbert cube Q is strongly infinite dimensional; let $Q = \prod_{k=1}^{\infty} I_k$ where $I_k = [-1, 1]$, let the projections be denoted by $\pi_k : Q \longrightarrow I_k$, and let $A_k = \pi_k^{-1}(-1)$ and $B_k = \pi_k^{-1}(1)$ (see [H-W, p. 49]).

In 1965, Henderson [He-1], [He-2] constructed a strongly infinite dimensional compactum containing no n-dimensional ($n \geqslant 1$) closed subsets; the following theorem states that there exist strongly infinite dimensional compacta which do not contain any n-dimensional ($n \geqslant 1$) subsets.

THEOREM. *Every strongly infinite dimensional compactum contains a strongly infinite dimensional subcompactum which contains no n-dimensional ($n \geqslant 1$) subsets.*

The question of whether or not such examples existed recently had taken on particular importance since Kozlowski [Ko] had shown that if no such examples existed, then *CE*-mappings could not raise dimension.

In 1967, Bing [Bi] gave a simpler construction of a Henderson-type example. In 1974, Zarelua [Za] constructed Henderson-type examples using a different approach. Recently, L. Rubin, R. Schori and the author [R-S-W] developed an axiomatic approach for constructing Henderson-type examples; the examples announced in the above theorem are constructed using this axiomatic development.

The author gratefully acknowledges the importance of a seminar run jointly with L. Rubin and R. Schori during the fall of 1976 at the University of Oklahoma.

REFERENCES

[A-1] P. S. Aleksandrov, *Some results in the theory of topological spaces obtained within the last twenty-five years*, Russian Math. Surveys 15 (1960), 23–83.

[A-2]   P. S. Aleksandrov, *On some basic directions in general topology*, Russian Math. Surveys, 19 (1964), 1—39.

[Bi]   R. H. Bing, *A hereditarily infinite-dimensional space*, General Topology and its Relations to Modern Analysis and Algebra, II (Proc. Second Prague Topological Sympos., 1966), Academia, Prague, 1967, pp. 56—62.

[He-1]   D. W. Henderson, *An infinite-dimensional compactum with no positive-dimensional compact subsets—a simpler construction*, Amer. J. Math. 89 (1967), 105-121.

[He-2]   ———, *Each strongly infinite-dimensional compactum contains a hereditarily infinite-dimensional compact subset*, Amer. J. Math. 89 (1967), 122—123.

[H-W]   W. Hurewicz and H. Wallman, *Dimension theory*, Princeton Univ. Press, Princeton, N. J., 1941.

[Ko]   G. Kozlowski, *Images of ANR's*, Trans. Amer. Math. Soc. (to appear).

[R-S-W]   L. Rubin, R. Schori and J. Walsh, *New Dimension-theory techniques for constructing infinite-dimensional examples* (submitted).

[Za]   A. V. Zarelua, *Construction of strongly infinite dimensional compacta using rings of continuous functions*, Soviet Math. Dokl. 15 (1974), 106—110.

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF OKLAHOMA, NORMAN, OKLAHOMA 73019

Current Address: Department of Mathematics, University of Tennessee, Knoxville, Tennessee 37916