$C(T)^*$ is $l^1(\Gamma)$ for some infinite set Γ. For any Banach space X, X^* is flat if X is flat. If X^* is an $L^1(\mu)$-space, then X being flat is equivalent to X^* being flat, which is equivalent to X^* not being $l^1(\Gamma)$ for any Γ. An $L^1(\mu)$ space is completely flat if and only if it is $L^1[0, 1]$. If X is isomorphic to a flat space, then X has an *infinite supported tree* and neither X nor X^* has the Radon-Nikodým property. The use of "completely flat" has strong motivation, because of the following surprising facts: Let s be a spanning girth curve and p be a point of s. Then there is a unique supporting hyperplane H of $S(X)$ at p; p is an interior point of a subset G of $H \cap S(X)$ whose closed affine span is H; for each q in G, $\sup\{\|q - r\|: r \in G\} = 2$; and G is the set of all $(p - q)/\|p - q\|$ for $q \neq p$ and $q \in s$.

ROBERT C. JAMES

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The theory of unitary group representations, by George W. Mackey, Chicago Lectures in Math., Univ. of Chicago Press, Chicago, Ill.,1976, x + 372 pp., $4.95.

It is probably impossible to write a comprehensive book on the theory of unitary representations. The subject, which logically begins in a modest way with complex representations of finite groups, proceeds to general compact groups, and goes on to treat a variety of noncompact groups, is simply too vast. By this time, as a result of the enormous activity in representation theory which began in the late forties and continues unabated, in fact exponentially, to this day, its sometimes alarming and ubiquitous role in a diversity of fields is well established. What is not well established is any agreement about what part or parts of the theory are the most important or how the subject should be organized or presented. At the same time there are disagreements about what open questions should be pursued and the future development of the theory. This naturally causes difficulties for anyone trying to write about representations. The reviewer sometimes envisages the appearance of a new book entitled, *What everyone ought to know about representations* and hordes of representers eagerly rushing out to acquire it, and later returning, disillusioned or angry with what they have found. Authors should also keep in mind that it is probably more difficult for an outsider to learn a substantial segment of representation theory than it is to write about it sensibly. This particular point is admirably put in the forward to Lang's recent book on $SL(2, R)$ in which he states, "It is not easy to get into representation theory, especially for someone interested in number theory, for a number of reasons. First, the general theorems on higher dimensional groups require massive doses of Lie theory. Second, one needs a good background in standard and not so standard analysis on a fairly broad scale. Third, the experts have been writing for each other for so long that the literature is somewhat labyrinthine." This statement is also significant in view of its tacit bias: the general theorems of the subject are either about representations of Lie groups or require some form of Lie theory in their understanding, a point with which the reviewer has considerable sympathy, but surely an indefensible one. The theory of unitary representations as developed in the first part of the book under review applies to general locally compact groups and makes no use of Lie theory. For this reason a more apt title would have been *On the general theory of unitary group representations*.

Despite the difficulties outlined above, the breadth, diversity, and unfinished aspects of representation theory are certainly to a large extent responsible for the basic appeal and vigor of the subject. In particular, the extraordinary scope of its applications is always a source of amazement. One might, for example, understand and take as a matter of course the existence of strong connections between harmonic analysis, several complex variables, special functions, and representation theory, yet find it psychologically difficult to imagine that it has any real connections with anything as mysterious as physics.

On a priori grounds it is also difficult to imagine a mathematical theory with serious applications to both physics and number theory. Of course, the most obvious way out of such impasses, is to admit, at least temporarily, that there is very little connection between p-adic representation theory and physics. The next logical step would be to admit that representation theory is a diverse subject with a number of fairly distinct and quite legitimate branches. If this principle were established it would be much easier to understand and tolerate a number of natural phenomena, e.g., why a "semisimple person" might feel somewhat akin to a fish out of water at a meeting of "solvable persons" and vice versa, or why some analysts are not entirely satisfied with the current constructions of *all* discrete series, etc..

With regards to exposition, this suggests that perhaps there ought to be a series of introductory monographs each dealing with a part of the subject and from a point of view appropriate for a given clientele. That would alleviate a number of problems, but undoubtedly cause some others, and in the end be unsatisfactory. After all, at some point in the future, there should be a sensible, less technical, unified theory of representations. In any case, this goal seems beyond reach at the present time.

One can hope, however, to define and explicate a substantial part of the common central core of the subject. There is no doubt that the construction of representations has been of basic importance. Many of the representations known at the present time are obtained by unitary induction or by some fairly elementary variant of that procedure. On the other hand, there are exceptions, for example, the nonunitary uniformly bounded representations constructed by the reviewer and E. M. Stein and the nonholomorphic discrete series first constructed by Narasimhan, Okamoto, and Schmid. Although the two exceptions just mentioned are important for harmonic analysis on semisimple Lie groups, the details of the constructions are quite complicated and too technical to fall into the category of what everyone should know. In contrast to this situation, the theory of unitary induction has a simplicity elegance, and general applicability which puts it in the category of basic material.

Mackey's major contributions to the theory of induced representations and his dominant role in the development of the subject are of course well known. The reviewer was first exposed to this material in 1955 in a series of lectures by Mackey during the summer quarter at the University of Chicago. The necessarily brief lectures covering global representation theory, direct integral decompositions, and induced representations were written up by Michael Fell and David Lowdenslager. This new book consists of two parts, the first being the original Chicago notes with minor corrections and clarifications, the second being a survey of early history and a long yet necessarily incomplete sketch of developments since 1955.

Unfortunately the second part which is nearly as long as the first has no table of contents. The only way to know what it contains or to find anything is to thumb through the pages searching for section headings. This is a definite nuisance. There is another irritating peculiarity; numbers 1 through 150 of the extensive 356 item bibliography for the second part appear elsewhere in a 1963 survey article in the Bulletin. These defects should be easy to remedy. Perhaps the University of Chicago Press can be persuaded to issue recall notices?

The lengthy sketch of developments after 1955 is divided into 14 sections. To mention just a few of its topics, it treats extensions of Mackey's own work such as the imprimitivity theorem, normal subgroup analysis and so on, as well as the state of unitary representation theory for nilpotent and solvable Lie groups, harmonic analysis, connections of representation theory with number theory, and applications to physics. These topics are discussed and interpreted from the point of view of the general theory presented in the notes. Thus, harmonic analysis is viewed as being essentially the problem of decomposing group actions on function spaces and more specifically as a problem of decomposing certain induced representations; the Selberg trace formula is seen as a special case of a Plancherel formula for representations induced by representations of discrete subgroups where the quotient is compact.

One might well argue that this perspective is too narrow, i.e., that the material in the notes is either not appropriate or not adequate for a number of topics, in particular, that it is not very helpful in certain questions concerning representations of semisimple Lie groups. For example, it is perhaps desirable but unlikely that global methods such as semidirect product analysis will succeed in replacing infinitesimal arguments and the subquotient theorem of Harish-Chandra in the analysis of the irreducible representations of semisimple Lie groups. But in any case, it makes for very interesting reading, often provides new insights, and presents an important unified view of representation theory. More to the point, however, the second part of the book admirably serves to fulfill its intended purpose, namely, that of placing the original and by now classic material in the 1955 lectures in a present day context.

RAY A. KUNZE

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Geometric algebra over local rings, by Bernard R. McDonald, Dekker, New York and Basel, 1976, xii + 421 pp., $29.50.

The name *geometric algebra* stems from E. Artin's book of that title [A] and