between complete metric spaces extends to a homeomorphism of two G_δ subsets.

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Dimension theory of general spaces, by A. R. Pears, Cambridge University Press, London, New York, Melbourne, Cambridge, 1975, xii + 428 pp., \$47.50.

Interest in making the concept of dimension mathematically rigorous probably began in 1890 with the appearance of an example due to Peano of a continuous map of the unit interval onto a triangle and its interior. This created the uneasy possibility that perhaps two Euclidean spaces of different dimensions might be homeomorphic. It is hard to imagine what mathematics might have been if this had turned out to be the case. It was a close call! Fortunately, L. E. J. Brouwer gave a proof in 1911 that if R^n and R^m are homeomorphic, then $n = m$. However, it was not until the 1920's that a topological theory of dimension began to be developed. The work of K. Menger and P. Urysohn as well as others brought into existence an elegant theory of dimension applicable to all separable metric spaces. It was only incidental to this theory that Euclidean n-space was n-dimensional. In true mathematical tradition, if the unthinkable had happened, dimension theory would have continued with the same fervor. The force of mathematical inquiry would have developed a mathematical structure similar to what we have today, except for the unfortunate footnote that Euclidean n-space is not n-dimensional! Mathematics would have suffered, but not dimension theory.

In 1928 the first text in dimension theory appeared, *Dimensionstheorie* by K. Menger. This book has historical value. It reveals at one and the same time the naïveté of the early investigators by modern standards and yet their remarkable perception of what the important results were and the future direction of the theory. Copies are difficult to obtain now, but it is worth the effort.

In 1941 the classic text *Dimension theory* by W. Hurewicz and H. Wallman appeared. This is one of most elegant treatises on any mathematical subject to appear. The presentation has an unmistakable clarity and an enviable economy of words and concepts. It has broad appeal to mathematicians. To a large extent this was because the authors made a deliberate attempt to pare away that which was purely technical and would be of interest only to specialists. The authors must have experienced psychological trauma relegating some of their own best results to footnotes where they would be found and savored only by the discriminating eye. But all who use this slender volume will be grateful.

Every significant area of mathematical research deserves monographs which detail its technical advances. Dimension theory has had several such texts in recent years. *Modern dimension theory* by J. Nagata (1965) treats the advances of the theory in general metric spaces. The significant breakthrough was the theorem of A. H. Stone in 1948 that a Hausdorff space is fully normal if and only if it is paracompact. Stone's theorem implies that every metric space is paracompact. M. Katětov and K. Morita made use of this to develop dimension theory for general metric spaces. *Dimension theory* by K. Nagami (1970) attempts to summarize developments in dimension for more general spaces. The theorems in Nagami's book require rather technical statement and it was intended mainly for the audience of dimension theorists and general topologists interested in having a handy summary of recent results and examples.

There are two recent texts to appear. P. S. Aleksandrov and B. A. Pasynkov's *Introduction to dimension theory* (in Russian, 1973) is an attempt to give a complete view of dimension theory from the general topological standpoint. The classical theory for separable metric spaces is covered and ultimately the reader is led in the book's 575 pages to the latest developments. The book by Pears is the most recent addition and has the same goal as Aleksandrov and Pasynkov.

Dimension theory of general spaces is an excellent reference for the dimension theorist and general topologist. Virtually every significant area of research in dimension theory is treated in depth. The classical theory for separable metric spaces and the now standard theory for general metric spaces is also included. The first two chapters are an introduction to general topology so that the book is completely self-contained.

An excellent feature of the text is its inclusion of a number of significant examples which have helped to shape dimension theory in recent years. Prabir Roy's construction of a metric space X which has ind $X = 0$ and Ind $X = 1$ is given detailed treatment. This example was important in bringing the theory of dimension for metrizable spaces to a satisfactory conclusion. The work of Katětov and Morita showed that Ind and dim are the same on metrizable spaces. It would have been simply an unacceptable state of affairs not to know whether all three dimension functions agree on metrizable spaces as they do on separable ones. Roy's example removes all doubt. The reader will find it rewarding to work it carefully through.

There are also examples of the pathological behavior of dimension on bicompacta. An example due to V. V. Filippov is presented as well as ones due to Lokucievskii and Vopenka. Filippov constructed a bicompactum X which has $\dim X = 1$, $\operatorname{ind} X = 2$, and $\operatorname{Ind} X = 3$. This example and its modifications show that except for the inequalities $\dim X \leqslant \operatorname{ind} X \leqslant \operatorname{Ind} X$, dim, ind, and Ind are independent variables on bicompacta.

One will find a section on local dimension containing Dowker's classic Example M of a completely regular space of covering dimension 1 and local dimension 0. There are sections detailing the results on dimension raising and lowering mappings, on the dimension of product spaces, on analytic dimension of algebras of continuous real-valued functions (reminiscent of Chapter 16 of Gillman and Jerison's *Rings of continuous functions*), and on dimension and bicompactification of completely regular spaces. There are extensive historical notes at the end of each chapter which help the reader put individual results in perspective. In these one is also introduced to still other advances and open problems.

The book does not treat homological dimension theory except in the historical notes. A separate volume would be necessary to handle this area with the same thoroughness. Rumor has it that such a volume is forthcoming from the Russian school. This would be a welcome addition. The theory for separable metric spaces is not treated with the thoroughness of Hurewicz and Wallman although there are some recent results appearing in the notes that bring one up to date. For those whose interest is separable metric spaces Hurewicz and Wallman remains the recommended text. One might criticize omission of some detail in this or that section of the book. Be reasonable! How long do you want the book to be? The historical notes and references will lead the reader to most additional results.

We have in Pears an excellent reference. The broad spectrum of recent advances is painted with a fine brush. Important (and complicated) examples are thoroughly examined. In a book of this level the statement of many theorems is necessarily technical. The novice may not appreciate the years of agonizing effort made by dimension theorists to weaken each hypothesis and make each theorem the paragon of precision. However, researchers who need the exact results of dimension theory for general spaces will find them here.

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Elementary calculus, by H. Jerome Keisler, Prindle, Weber and Schmidt, Boston, 1976, xviii + 880 + 61 (appendix) pp.

Our educational system contains many interesting paradoxes. We tell the students to get involved in the world, and the curriculum becomes increasingly abstract. Courses in sociology, anthropology, economics, et cetera are intro-