completely covers the various special cases, it gradually became clear that it gives for many important questions either no answers at all or very inadequate answers. For example, this is the case concerning the asymptotic behavior of eigenvalues and eigenfunctions. Moreover, one of the basic theorems of the entire abstract theory is the spectral decomposition of an operator in terms of a so-called resolution of the identity. For differential operators this spectral decomposition can usually be described by means of the solution of an appropriate equation. Thus in questions regarding the specific description of the resolution of identity, the general spectral theory is of very little help. This perhaps explains the paradoxical fact that a few years after the actual completion of the abstract spectral theory of selfadjoint operators, intensive work on the spectral theory of selfadjoint differential operators began.

For the convenience of the reader who is not familiar with abstract spectral theory, Chapter 13 discusses concisely this theory without proofs and indicates various connections with the spectral theory of differential operators. Also necessary results from analysis that are used in this book are given in Chapter 14 as a ready reference.

The material of the book is very well organized and the proofs are clear. It is an excellent source for anyone who wants to learn or review the essentials of the subject. It is a valuable and welcome addition to the literature.

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Logic in algebraic form, by William Craig, Studies in Logic and the Foundations of Mathematics, vol. 72, North-Holland Publishing Company, Amsterdam and London, 1974, 204 + viii pp., \$18.50.

An algebraic approach to non-classical logics, by Helena Rasiowa, Studies in Logic and the Foundations of Mathematics, vol. 78, North-Holland Publishing Company, Amsterdam (also, PWN-Polish Scientific Publishers, Warszawa) 403 + xv pp., 1974.

The two volumes under review belong in spirit to the classical period of algebraic logic which is the study of logical problems by algebraic means and of algebraic structures arising in mathematical logic. They do not use the tools of categorical logic which is the modern inheritor of the subject. They are concerned with propositional and first order languages, theories expressed in them and algebraic structures derived from them.

Craig's approach to algebraic logic is highly personal and, although it links up with the theory of polyadic and cylindric algebras, it has its own orientation and very distinctive flavour. *Logic in algebraic form* is thought provoking and worth studying from the philosophical, proof-theoretic and algebraic viewpoints. Should such formulas as $P(v_0)$ and $(v_1 = v_1) \wedge P(v_0)$ be identified in an algebraic formulation of first order logic? They are, of course, provably equivalent and so would be identified in the cylindric or polyadic algebra setting. They are not identified, say, in Lawvere's categorical logic approach to elementary theories. Craig argues that they should not because they do not determine the same operation, that of the first formula depending on v_0 only whereas that of the second depends also on v_1. So the matter hinges on how one defines the operation $\vdots\varphi\vdots$ associated to a formula φ.

The starting point of Craig's book is the assertion that the concern of first order logic is precisely these operations $\vdots\varphi\vdots$. For a given φ and nonempty universe U, $\vdots\varphi\vdots$ assigns to appropriate denotations of the predicate variables the n-ary relation $X \subset^n U$ determined by φ on U as value, n being related to the variables occurring free in φ. It is argued that amongst the seemingly equivalent definitions of $\vdots\varphi\vdots$ only one is the correct choice and that ordinary first order logic itself, and hence also cylindric and polyadic algebras, are ill-equipped for the theoretical study of these operations mostly because the provability of a formula $\varphi \leftrightarrow \psi$ implies the equality of the operations corresponding to φ and ψ only when an extraneous nonformalizable comment such as "the first individual variable not occurring free in φ is the same as the one for ψ" is available. The book is concerned mostly with the development of languages termed algebraic, or better "equational", more suited for the study of the objects $\vdots\varphi\vdots$ from this point of view and, also but secondarily, with algebraic structures stemming from such languages.

Amongst several competing candidates in the choice of an operation $\vdots\varphi\vdots$ for the formula φ, the author retains the one which makes the value X of $\vdots\varphi\vdots$ an n-ary relation where $n = \bar{\varphi}$ is the least r such that $i < r$ for each individual variable occurring free in φ. Here, $v_0, v_1, v_2, \ldots$ are the individual variables of the first order language concerned. Interesting but debatable philosophical arguments which, despite the author's advice to the contrary, should not be skipped, disqualify the other claimants which assign to $\vdots\varphi\vdots$ instead of X such values as the set of restrictions to the variables actually occurring free in φ of functions in X, or else, the set of infinite sequences which extend n-sequences in X. Such candidates are guilty of yielding as values for $\vdots\varphi\vdots$ objects which, for example, are not arguments for such operations and, therefore, make it impossible to relate directly the mode of composition of $\vdots\varphi\vdots$ to the syntactical structure of φ. The arguments and values of the operation $\vdots\varphi\vdots$ belong to finite cartesian powers ${}^nU$ of the domain U and, hence, in general, to different sets. Therefore substitutions of values of such operations as arguments are plagued with the usual unpleasant, though relatively trivial, realities of substitution theory in algebraic logic. A fruitful and original idea of the book is to replace the operations $\vdots\varphi\vdots$ by operations on the set ${}^{|\omega}U$ of all finite sequences of elements of U or on the infinite cartesian power ${}^{\omega}U$. This is achieved through very carefully selected mappings.

Having found some generators for the sets of operations generated by the new $\vdots\varphi\vdots$'s under composition, we are led to languages having as alphabet, function symbols standing for these generators, the equality sign and variables of one kind only which are intended to vary on the set of all subsets of ${}^{|\omega}U$ or on the set of all subsets of ${}^{\omega}U$. Of course, only equations are expressible in these languages. One complete and sound axiomatisation of the set of valid equations is presented in the first case and two in the second case. To first order formulas φ and ψ correspond terms τ and ρ in such an algebraic language and the objective is met that the equality $\tau = \rho$ holds if and only if $\vdots\varphi\vdots = \vdots\psi\vdots$ or just about.

The first (and second) set of generators mentioned above consists, in addition to the Boolean operations, of a diagonal element D, cylindrifications C_i, a certain projection P and its conjugate Q. Therefore the Boolean algebras $\mathcal{P}(^{|\omega}U)$ and $\mathcal{P}(^{\omega}U)$ underlie augmented cylindric algebra-like structures which arise in the study of models of (equational) theories equivalent to first order theories and which are formulated in the new languages. These augmented structures are given two characterizations due to Charles M. Howard in the only truly algebraic chapter of the book.

The third set of generators employs the substitution operator $S\alpha$ which is an inverse image operator, and also a direct image operator $T\alpha$. The author has shown elsewhere [see A. Daigneault, ed., *Studies in algebraic logic* (Math. Assoc. Amer., 1974)] that the axiomatisation obtained is equivalent to the theory of polyadic algebras with diagonal elements. This shows that it is possible to give priority to transformation operators over quantifiers contrary to what is done in the theory of cylindric algebras.

The book by Rasiowa is a rather encyclopedic companion to *The mathematics of metamathematics* (Warsaw, 1963) by H. Rasiowa and R. Sikorski. Its subject matter essentially encompasses that of this forerunner which dealt mainly with the algebraic approach to classical and intuitionistic logics at the propositional and first order levels. Here one is interested in a wider class of logics mainly at the propositional level though a supplement discusses the first order level also.

The book's algebraic approach follows the classical line of Lindenbaum and Tarski which consists in associating to a theory formalized in a given logic its algebra of classes of formulas equivalent modulo that theory. Concentrating on the axiomatisation of such algebras at the propositional level, this idea leads to the introduction of a whole panoply of algebras such as: implicative algebras, distributive lattices, quasi-Boolean algebras, relatively pseudo-complemented lattices, pseudo-Boolean algebras, quasi-pseudo-Boolean algebras, topological Boolean algebras, Post algebras, etc.

To these correspond so many logics which all fall within the author's concept of implicative extentional calculi which serves as unifying theme. Amongst them one finds some, such as positive implicative logic, classical implicative logic, minimal logic with seminegation and constructive logic with strong negation, whose algebraic treatment seems to appear here for the first time in book form. Because of its quasi-exhaustiveness as to the class of logics treated, *An algebraic approach to non-classical logics* will be an excellent reference work for the definitions of these various logics, algebras and their interrelations. However, the philosophical motivation for these concepts is disregarded.

The supplement on first order algebraic logic is somewhat disappointing, the emphasis being put on generality rather than depth. It achieves a uniform treatment of the concept of validity and of the completeness theorem for the first order logics extending the propositional logics considered in the book through a generalization of the famous algebraic proof of Rasiowa and Sikorski of the completeness theorem of classical first order logic and of the subsequent concept derived from that proof of the canonical realization of an elementary theory. The supplement does not touch upon the theory of cylindric and polyadic algebras nor does it tackle with algebraic means any part of model theory.

An excellent article by the same author summing up much of the material of the book and developing further the first order theory of multi-valued logic has appeared in *Studies in algebraic logic* referred to above.

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On measures of information and their characterizations, by J. Aczél and Z. Daróczy, Mathematics in Science and Engineering, vol. 115, Academic Press, New York, San Francisco, London, 1975, xii + 234 pp., $24.50.

The purpose of the authors cannot be stated more clearly than in the following lines of the preface (p. XI):

"We shall deal with measures of information (the most important ones being called entropies), their properties, and, reciprocally, with questions concerning which of these properties determine known measures of information, and which are the most general formulas satisfying reasonable requirements on practical measures of information. To the best of our knowledge, this is the first book investigating this subject in depth". In fact, from the 234 pages of the book, only 6 are devoted to simple applications to logical games (pp. 33 –38) and 17 to optimal coding (pp. 42–50 and pp. 156–164).

But, as the authors write (p. 29) "the problem is to determine which properties to consider as natural and/or essential". From the beginning they make a choice, which implies consequences of paramount importance: the measure of the information yielded by one event A depends only upon the probability $P(A)$ of this event; due to this choice they restrict themselves to the foundations of the classical information theory, initiated in 1948 by Claude Shannon and Norbert Wiener. Of course this classical theory has proved to be very useful indeed in many branches of science, its greatest success being the foundation and development of communication theory: the fundamental hypothesis means that the amount of information given by a message depends only on its frequency; very unexpected messages give considerable information. But, as has been often pointed, no account of the semantic content of the message, could be taken in this way. Moreover there is a subjective aspect of information, which is entirely out of the scope of the classical theory: the same event does not yield the same amount of information to all the observers.

These rather obvious remarks show that the classical information theory