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RESEARCH ANNOUNCEMENTS

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[*Note.* The criteria described above are new; in the transition period they will not necessarily be met by the announcements appearing in this issue.]

NONPOSITIVELY CURVED MANIFOLDS

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Let M be a simply connected, complete, Riemannian manifold of nonpositive sectional curvature. In [7], Eberlein and O'Neill have obtained a boundary $M(\infty)$ for M which is the set of asymptotic classes of geodesics in M. The limit set $L(G)$ of a group G of isometries is defined as the intersection with $M(\infty)$ of any orbit of G.

The following theorems may be considered as analogous results to several statements in Borel's density theorem for symmetric spaces [2].

THEOREM 1. *Let M be a simply connected, complete, Riemannian manifold of nonpositive sectional curvature and without Euclidean factor in its de Rham decomposition. Suppose that G is a subgroup of $I(M)$ and $L(G) = M(\infty)$. Then the centralizer $Z(G, I(M))$ is trivial. Moreover, either G has a proper, closed, invariant subset in $M(\infty)$ or G is semisimple.*

COROLLARY 1. *Let M be a complete, homogeneous, Riemannian manifold of nonpositive sectional curvature and without Euclidean factor in its de Rham decomposition. Then, either $I_0(M)$ has a proper, closed, invariant subset in $M(\infty)$*

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or $I_0(M)$ is a connected, noncompact, semisimple Lie group, that is, M is a noncompact symmetric space.

Let N be a complete Riemannian manifold of nonpositive sectional curvature. Consider the universal covering $\pi: M \longrightarrow N$ of N. Let G denote the group of deck transformations associated with this covering.

THEOREM 2. *Let N be a complete Riemannian manifold of nonpositive sectional curvature and without Euclidean factor in its universal covering manifold M. Assume that $L(G) = M(\infty)$ and G has no closed invariant subset in $M(\infty)$. Then either $I(M)$ is discrete or $L(I_0(M)) = M(\infty)$. If, in addition, $G \subset I_0(M)$, then $I_0(M)$ is a noncompact, connected, semisimple Lie group and N is a symmetric space form.*

The visibility axiom for complete, Riemannian manifolds of nonpositive sectional curvature can be found in [7].

THEOREM 3. *Let N be a complete, visibility manifold. Assume that $L(G) = M(\infty)$. Then either $I(M)$ is discrete or $I_0(M)$ is a noncompact, connected, semisimple Lie group.*

COROLLARY 2. *Let N be a complete, visibility manifold. If N has finite volume, then either $I(M)$ is discrete or $I_0(M)$ is a noncompact, connected, semisimple Lie group.*

COROLLARY 3. *Let N be a two-dimensional, complete, visibility manifold. Assume that $L(G) = M(\infty)$. Then either $I(M)$ is discrete or M is the hyperbolic plane, that is, N is a two-dimensional hyperbolic space form.*

Similar results have been obtained independently by Byers [3] and Heintze [8].

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