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Deterministic and stochastic optimal control, by Wendell H. Fleming and Raymond W. Rishel, Applications of Mathematics, vol. 1, Springer-Verlag, New York, Heidelberg and Berlin, 1975, 222 pp., $24.80.

The deterministic optimal control problem under consideration is the following. The state of a system evolves according to a system of first order ordinary differential equations whose right hand side is under our "control" and it is required to control the system so as to minimize a given functional of the control and state. This functional is usually called a performance index, following the engineering literature. In the stochastic optimal control problem the state is a finite dimensional diffusion which evolves according to a system of stochastic differential equations under our control, and it is required to control the system so as to minimize the expected value of a given performance index. The first half of the book deals with the deterministic problem; the second half deals with the stochastic problem.

The mathematical theory of deterministic optimal control is in a relatively complete and satisfactory state. The authors have focused their attention on three important and related aspects of this theory, which we shall discuss below. The stochastic theory is currently not as complete. The approach in this book is by way of dynamic programming. Much of the material was originally developed by the authors themselves and appears in book form for the first time.

There is considerable difference in the mathematical prerequisites for reading the two parts of the book. The first part should be accessible to anyone who has completed the standard first year graduate course in analysis. After a stage-setting initial chapter on the calculus of variations the authors focus on their three principal topics: the Pontryagin Maximum Principle (necessary conditions); existence theorems; and dynamic programming in relation to control problems. In their proof of the maximum principle the authors combine features of several different proofs to produce a very appealing proof of the theorem. The proofs of the existence theorems are essentially the original proofs of Filippov in the case of bounded controls and of Cesari in the case of unbounded controls. The dynamic programming chapter introduces some of the ideas used in the stochastic control problem and presents a sufficiency theorem in terms of the structure of the trajectory fields that one gets by "solving" the maximum principle. In the proof of the sufficiency theorem the specialist will appreciate the simplification achieved by the use of Federer's co-area formula.

The minimum mathematical prerequisite for part two is a beginning graduate course in probability which includes martingales. Knowledge of stochastic processes would also be useful. In Chapter V, which with the exception of one section is independent of the rest of the book, the authors give a crash course on those aspects of stochastic processes and related topics needed in the study of stochastic control problems. For some readers Chapter V will serve to fill in gaps in their backgrounds. For others it will serve as a good outline of the homework that they will have to do if they become serious about stochastic control theory.

As already noted, the stochastic control problems are treated via dynamic programming in a mathematically rigorous way. For the class of processes considered, the study of the optimal control problems is reduced to the study of certain second order nonlinear partial differential equations. The existence of solutions of the partial differential equations and the properties of the solutions are then investigated with considerable success.

The style is lean and clean. Proofs and major developments are broken up into easily digestible pieces. At appropriate places references are given to the literature for further development of topics or to alternate developments.

This is definitely a book that both the specialist and the person interested in an uncluttered introduction to some of the major aspects of deterministic and stochastic control will want to read and own.

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Combinatorial algorithms, by Albert Nijenhuis and Herbert S. Wilf, Academic Press, New York, San Francisco, and London, 1975, xiv + 253 pp., $19.50.

Combinatorial algorithms are computational procedures which are designed to help solve combinatorial problems. Combinatorial problems are problems involving arrangements of elements from a finite set *and* selections from a finite set. These problems can be divided into three basic types: (1) enumeration problems, (2) existence problems, and (3) optimization problems. In enumeration problems the goal is *either* to find how many arrangements there are satisfying the given properties *or* to produce a list of arrangements satisfying the given properties. In existence problems the goal is to decide whether or not an arrangement exists satisfying the given properties. In optimization problems the goal is to find where a given function of several variables takes on an extreme value (maximum or minimum) over a given finite domain. Graph theoretic algorithms are included in the above definition of combinatorial algorithms.

In this book Nijenhuis and Wilf discuss various combinatorial algorithms. Their enumeration algorithms include a chromatic polynomial algorithm and a permanent evaluation algorithm. Their existence algorithms include a vertex coloring algorithm which is based on a general backtrack algorithm. This backtrack algorithm is also used by algorithms which list the colorings of a graph, list the Eulerian circuits of a graph, list the Hamiltonian circuits of a graph and list the spanning trees of a graph. Their optimization algorithms include a network flow algorithm and a minimal length tree algorithm. They give 8 algorithms which generate at random an arrangement. These 8 algorithms can be used in Monte Carlo studies of the properties of random arrangements. For example the algorithm that generates random trees can be