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Methods of numerical mathematics, by G. I. Marchuk (Translated by J. Ružička), Applications of Mathematics, vol. 2, Springer-Verlag, New York, Heidelberg, Berlin, 1975, xii + 316 pp., $29.80.

The path leading from a mathematical continuum description of a scientific process, to a specific verifiable description or prediction of the results, can take unexpected and frustrating turns. The simplistic "put it on the computer" often surprises the proposer with an outcome of wildly gyrating numbers; but, less dramatically, the computer output may lack accurate detail, violate physical principles, and require many hours of expensive computer time to generate. That numerical mathematics is still not always able to cope satisfactorily with partial differential equations is due, in part, to the lack of recognition by many mathematicians that partial difference equations are at least as interesting and difficult as the differential equations they model.

On a mathematical level, difficulties with difference equations reflect the gap between discrete and continuous mathematics. For example, the Hille-Yosida-Phillips theorem says that if U is a closed, densely defined operator, then there exists a constant M such that $\|\exp(Ut)\| \leqslant M$ for $t \geqslant 0$ if and only if $\|(\lambda - U)^{-r}\| \leqslant M\lambda^{-r}$ for $\lambda > 0$, $r = 1, 2, \ldots$. For a bounded operator A on a Banach space, it is easy to show that $\|A^r\| \leqslant M$ implies $\|(\lambda - A)^{-r}\| \leqslant M(|\lambda| - 1)^{-r}$ for $|\lambda| > 1$, $r = 1, 2, \ldots$. However, it is not known (even for matrices) whether or not the resolvent inequality implies the existence of a constant N, *depending only on* M, such that $\|A^r\| \leqslant N$, $r = 1, 2, \ldots$ (see A. Gibson, *A discrete Hille-Yosida-Phillips Theorem*, J. Math. Anal. Appl. **39** (1972), 771–792, for some results on this problem). This result (true or false) is related to the stability theory of *linear* difference equations; the nonlinear theory has many deep, open questions: for example, does any commonly used difference approximation to the gas dynamics equations converge to a physically correct weak solution?

On a computational level, difficulties with difference equations could be blamed on the existence of many consistent difference schemes corresponding to a given partial differential equation. How is one to make a rational choice? Constraints on accuracy and efficiency are usually incompatible, mutually as well as with imposed physical conservation laws. There is as yet no unique best method for a given problem in mathematical physics. In the last section of the book being reviewed the author points out that "The optimization of numerical processes is presently, without any doubt, one of the central problems of computational sciences; it stimulates exploration of few numerical algorithms and methods of their realizations."

This very complex question of optimization is not attacked in Marchuk's book. Instead, "this book is primarily intended for the benefit of those encountering truly complicated problems of mathematical physics for the first time, who may seek help regarding rational approaches to their solution". Those encountering such problems will be helped by this book. I found it readable, with an excellent bibliography for those who wish to go more deeply into the subject, which is not all of numerical analysis, as the title

indicates, but primarily numerical methods for partial differential equations. Modern methods, for example, finite elements, fast Fourier transform, and the method of large particles (particle-in-cell method) are discussed or mentioned, while a large part of the book is devoted to the powerful splitting-up method. This method is based on the formal relations

$$e^{(A_1+A_2)\Delta t} = e^{A_1\Delta t}e^{A_2\Delta t} + O(\Delta t)^2$$

and

$$e^{(A_1+A_2)\Delta t} = e^{A_1\Delta t/2}e^{A_2\Delta t/2}e^{A_2\Delta t/2}e^{A_1\Delta t/2} + O(\Delta t)^3$$

which permit the problem $u_t = (A_1 + A_2)u$ to be solved as a sequence of simpler problems.

I do find one fault with this book. It paints too rosy a picture of computational physics. Only the most serendipitous practitioner will be able to use successfully some of the recommended methods on complicated problems. A better balance would have resulted with the inclusion of a chapter on ways of analyzing the effectiveness of a scheme; phase error analysis, operation counts, long-time stability properties, and detailed truncation error analysis. Such a chapter might also have included some numerical results to show the bad answers that some apparently good methods can produce.

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Funktionalanalysis, by Harro Heuser, Mathematische Leitfäden, B. G. Teubner, Stuttgart, 1975, 416 pp.,

Geometric functional analysis and its applications, by Richard B. Holmes, Graduate Texts in Mathematics, No. 24, Springer-Verlag, New York, Heidelberg, Berlin, 1975, x + 246 pp., $16.80.

Functional analysis, by Michael Reed and Barry Simon, *Methods of modern mathematical physics*, vol. I, Academic Press, New York and London, 1972, xvii + 325 pp., $13.50.

Methods of modern mathematical physics, vol. II, *Fourier analysis, self-adjointness*, by Michael Reed and Barry Simon, Academic Press, New York, 1975, xv + 361 pp., $24.50.

These are three quite different introductions to functional analysis, addressed to different constituencies; all three are intended for use as graduate level textbooks, with varying demands on the reader's mathematical background. Heuser's book is appropriate for general mathematics students as well as future specialists. Holmes' book stresses Banach spaces and applications to optimization theory. Reed and Simon's series (apparently projected for at least five volumes) is an exposition of functional-analytic methods in modern mathematical physics. In different ways, these books are all written admirably, but I confess that for sheer craftsmanship and pedagogical judgement, my heart belongs to Heuser.