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INTEGRABILITY OF SHIFTED TRIGONOMETRIC SERIES

BY JOHN W. GARRETT AND ČASLAV V. STANOJEVIĆ

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Abstract. We give generalizations and extensions concerning integrability of shifted and weighted trigonometric series of Boas and Rees-Stanojević.

Boas [1] proved the following two integrability theorems for certain weighted sine and cosine series.

THEOREM A. *Let* $g(x) = \Sigma_{n=1}^{\infty} b(n)\sin nx$ *where* $b(n)$ *decreases to zero. Then for* $0 \leqslant \gamma \leqslant 1$, $x^{-\gamma}g(x) \in L[0, \pi]$ *if and only if* $\Sigma_{n=1}^{\infty} n^{\gamma-1}b(n) < \infty$.

THEOREM B. *Let* $f(x) = \Sigma_{n=1}^{\infty} a(n)\cos nx$ *where* $a(n)$ *decreases to zero. Then for* $0 < \gamma < 1$, $x^{-\gamma}f(x) \in L[0, \pi]$ *if and only if* $\Sigma_{n=1}^{\infty} n^{\gamma-1}a(n) < \infty$.

Recently Rees and Stanojević [2] proved a similar theorem for a shifted sine series.

THEOREM C. *Let* $g(x) = \Sigma_{n=1}^{\infty} b(n)\sin(n + \frac{1}{2})x$ *where* $b(n)$ *decreases to zero. Then* $x^{-1}g(x) \in L[0, \pi]$ *if and only if* $\Sigma_{n=1}^{\infty} b(n) < \infty$.

Theorem C is a by-product of an integrability theorem for certain cosine sums introduced in [2]. It follows after summation by parts of these cosine sums due to the form of the Dirichlet kernel. This paper gives extensions of Theorems A and B in the direction indicated by Theorem C.

THEOREM 1. *Let* $g(x) = \Sigma_{n=1}^{\infty} b(n)\sin(n + \alpha)x$ *where* $b(n)$ *decreases to zero and* $0 \leqslant \alpha \leqslant \frac{1}{2}$. *Then for* $0 \leqslant \gamma < 1, x^{-\gamma}f(x) \in L[0, \pi]$ *if and only if* $\Sigma_{n=1}^{\infty} n^{\gamma-1}C(n) < \infty$.

THEOREM 2. *Let* $f(x) = \Sigma_{n=1}^{\infty} a(n)\cos(n + \alpha)x$ *where* $a(n)$ *decreases to zero and* $0 \leqslant \alpha \leqslant \frac{1}{2}$. *Then for* $0 < \gamma < 1, x^{-\gamma}f(x) \in L[0, \pi]$ *if and only if* $\Sigma_{n=1}^{\infty} n^{\gamma-1}a(n) < \infty$.

Proofs and details of these theorems will appear elsewhere.

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF MISSOURI-ROLLA, ROLLA, MISSOURI 65401

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