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WHITEHEAD THEOREMS IN PROPER HOMOTOPY THEORY

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Following Chapman [3] we define a continuous map $f: X \longrightarrow Y$ to be *proper* iff for each compactum $B \subset Y$ there exists a compactum $A \subset X$ such that $f(X \backslash A) \cap B = \emptyset$. (This is just a reformulation of the usual notion of a proper map.) Then maps $f, g: X \longrightarrow Y$ are said to be *weakly properly homotopic* iff for each compactum $B \subset Y$ there exists a compactum $A \subset X$ and a homotopy (dependent on B) $F = \{F_t\}: X \times I \longrightarrow Y$ (where $I = [0, 1]$) such that $F_0 = f$, $F_1 = g$, and $F((X \backslash A) \times I) \cap B = \emptyset$. If, in fact, there exists a proper map $F: X \times I \longrightarrow Y$ which satisfies $F_0 = f$ and $F_1 = g$, then we say that f and g are *properly homotopic*. The notions of *weak proper homotopy equivalence* and *proper homotopy equivalence* are now defined in the obvious way.

In [7, pp. 489–491] Siebenmann obtained various convenient criteria for a proper map of locally finite simplicial complexes to be a proper homotopy equivalence. Siebenmann's proof seemed to require a finite dimensional assumption. Later, E. Brown [2, p. 34], and Farrell, Taylor and Wagoner [6] claimed to be able to remove the finite dimensional assumption. In [4] we give an example, using an interesting map of J. F. Adams [1], which shows that the finite dimensional assumption *is* necessary. On the positive side, we prove in [4] the following useful (see [5]) Whitehead type theorem.

THEOREM. *Let $f: X \longrightarrow Y$ be a proper map of locally finite simplicial complexes such that f is a weak proper homotopy equivalence. Then f is weakly properly homotopic to a proper homotopy equivalence.*

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