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# RESEARCH ANNOUNCEMENTS

## A THEOREM ON THE RANK OF A DIFFERENCE OF MATRICES

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There have been several results on the characterization of $\operatorname{rank}(A - S) = \operatorname{rank}(A) - \operatorname{rank}(S)$ and $\operatorname{rank}(A - S) = \operatorname{rank}(A)$. A result that characterizes some intermediate cases is

THEOREM. *Let $S = UV^H$ where $U$ and $V$ are any matrices such that $AA^+U = U$ and $V^HA^+A = V^H$. Then* $\operatorname{rank}(A - S) = \operatorname{rank}(A) - p$, $0 \leqslant p \leqslant k = \operatorname{rank}(S)$, *where* $p = \operatorname{nullity}(V^HA^+U - I)$.

This theorem gives as a special case the Wedderburn-Householder-Funderlic result [1].

COROLLARY. *If $U$ and $V$ have full column rank, then the equality* $\operatorname{rank}(A - UV^H) = \operatorname{rank}(A) - \operatorname{rank}(UV^H)$ *holds if and only if there are matrices $X$ and $Y$ such that $U = AX$ and $V = A^HY$ with $Y^HAX = I$.*

### REFERENCE

1. R. E. Funderlic, *Norms and semi-inverses*, Union Carbide Corp., Nuclear Division Report CTC-35, 1970, Dissertation presented to the University of Tennessee under the direction of A. S. Householder.

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