General systems theory: Mathematical foundations, by M. D. Mesarovic and Y. Takahara, Mathematics in Science and Engineering, Vol. 113, Academic Press, New York, 1975, xii +268 pp., $20.00.

Historically, serious mathematical research on systems theory is traceable to solutions of difficult physical problems in dynamics connected with so-called systems of the world based on efforts by Ptolemy, Copernicus, Kepler, Galileo, Newton, Euler, Laplace, and Gauss, among others. These scientific investigations provide an interesting mixture of knowledge for its own sake and knowledge for the sake of commerce in the form of improved navigational techniques. In more recent times, systems theory, orienting itself still more with technology, has been influenced by physico-mathematical inquiries underlying the operation and design of hardware associated with power-plant governors (J. C. Maxwell); the position control system of the steering engine of ships (N. Minorsky); extrapolation servomechanisms and other cybernetic systems (N. Wiener); communication systems, secrecy systems, and digital switching systems (C. E. Shannon); general linear filters (R. E. Kalman); and adaptive or self-organizing control systems (W. R. Ashby, R. E. Bellman and L. A. Zadeh).

The purpose of the research monograph under review is to provide a unified and formalized mathematical approach to all major systems concepts. Neither practical applications nor philosophical ramifications of general systems theory are discussed. The authors promise to present these elsewhere. Special attention is devoted to formal aspects of deterministic input-output systems. Learning systems, decision-making systems, and goal-seeking systems, *per se,* are discussed only incidentally in the appendices. The point of entry for the authors' development of a general systems theory is the identification of a *system* with a set-theoretical relation. This approach certainly has the feature of generality and abstractness to it. With this degree of generality and abstraction one might expect, and rightly so, little content in the results. Indeed, the authors prove little that does not depend on the use of more formidable algebraic structure for the sets; e.g., that the sets are linear spaces over the same ground field. However, by adding algebraic structure prematurely, the authors miss a wonderful opportunity to apply the deep mathematical theory of ordinal relations (not necessarily finite), as developed by C. S. Peirce, E. Schröder, and A. Tarski, to the theory of general systems. The authors profess to develop an *axiomatic* approach to systems theory based on set-theoretical concepts. Nevertheless, they fall short of this desirable goal on two counts. First, they could have reduced their prime notion of a *relation* to a purely set-theoretical concept (say, by using Wiener's definition of a relation or Kuratowski's modification) and, then, used a suitable axiomatic set theory to underpin their whole edifice. Thus, a system becomes a set of sets (the notion of a relation being redundant) in, say, ZF set theory. This proposal has a hidden bonus: it more completely aligns systems theory with an axiomatic set theory; a link that might have pleased Minkowski, but saddened Hardy. Then it becomes possible to interpret independence results for axiomatic set theory (see, e.g., P. J. Cohen, *Set theory and the continuum hypothesis*, Benjamin, New York, 1966 or J. B. Rosser, *Simplified independence proofs*, Academic Press, New York, 1969) in the context of modern dynamical systems. In the reviewer's opinion, independence results for set theory have applications to physical theory with potential impact on society at least as great as those applications of independence results for classical geometry.

Linear systems occupy a central position in the authors' theory. Indeed their deepest results deal with these systems, which have a global response function with an additive representation whose summands are linear mappings on linear spaces over the same ground field. The authors' proof of this result uses Zorn's lemma twice! Surely there must be a *constructive* proof of this basic result; i.e., a proof that allows one to construct the required response function. Although classical realization theory for networks and linear feedback control systems depends heavily on concepts from analysis (e.g., positive real functions, orthogonal functions and Hurwitz polynomials), the authors' realization theory is more oriented toward algebraic notions (e.g., commutative diagrams and semigroups). Indeed, the algebraic approach pervades the methodology of the authors. To this extent, the book is far more a sequel to Ashby's approach to cybernetical systems than to Wiener's equally important (and somewhat deeper) analytical approach. This correlative comment is based on the reviewer's attendance at several of their formal lectures and seminars as well as on his reviews of their independent cybernetical investigations; e.g., MR **22** #4569 and MR **23** #B1040.

The penultimate Chapter 11 on computability, consistency and completeness is less clear to this reviewer than some of the other more polished chapters. The authors subsume all computational systems under systems which reach an equilibrium state in finite time. Note that there is no effective way of deciding for an arbitrary system whether or not it has reached an equilibrium state. The authors' interesting analogue of *Gödel's first incompleteness theorem* (pp. 214–215) is not altogether convincing since there is no mention of either effectivity of processes or the specific structure of the system K which makes it either inconsistent or incomplete. E.g., the result seems to fail in case K is the monadic functional calculus of first order or K is the system of well-formed formulas in Skolem normal form for satisfiability in the pure first-order functional calculus. Indeed, it is not apparent whether the authors' argument produces a recursively generated logic. On the other hand, E. L. Post's rigorous, beautiful, and elegant "miniature" proof (Bull. Amer. Math. Soc. **50** (1944), 284–316) of Gödel's theorem based on the theory of recursively enumerable sets would easily fit into two pages and enhance the value of any treatment of general logical systems. Further, the authors appear to have missed an excellent opportunity to exploit P. R. Halmos' deep results on *Algebraic logic*, Chelsea, New York, 1962 for the study of logical systems and on the related *algebraic* version of logical completeness: every polyadic algebra is semisimple.

The authors' most significant achievement is the *systematic* application of ideas of elementary category theory to systems theory; not that this approach to systems study is new. E.g., such ideas are used by Eilenberg (*Automata, languages and machines*, Academic Press, 1975), Arbib and Manes (*Arrows, Structures, and Functors*, Academic Press, 1975), Dididze (Russian), and Bautor (German) for the study of logical systems. The authors of the monograph under review provide elegant, if not tightly reasoned, arguments in this area. On the other hand, there is an undesirable trend in this kind of enterprise. Researchers seem to insist on the development of category-theoretic abstractions of the notion of recursivity rather than developing a "recursive category theory" which begins with a recursively enumerable class of objects together with various recursive functors. Use of recursion-theoretic ideas is not likely to enfeeble the wings of that soaring eagle, category theory. In any case, Von Neumann has warned us all about the adverse things that can happen to the fabric of the mathematical sciences as our theories, governed by aesthetical desiderata alone, recede further and further from contact with physical reality.

In conclusion, this fine work is only a first try at the much desired paradigm for systems theory. Axiomatization is never achieved and much in the overview is left unanswered (e.g., (1) how is the structure of nonlinear systems clarified and extended by the authors' approach, and (2) what *new* systems, if any, are predicted by the present approach). Nevertheless, this carefully written and attractively reproduced treatise passes the BUNTSI test for scientific publishability: much of the material is *B*eautiful, *U*seful, *N*ew, *T*rue, *S*erious, and *I*nteresting.

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An introduction to invariant imbedding, by R. Bellman and G. M. Wing, John Wiley & Sons, New York, 1975, xv+250 pp., $18.95.

This book gives an introduction at an elementary level to a method for solving boundary value problems in one independent variable. The method is called invariant imbedding and sometimes it is also referred to as the method of continuation. The idea is to let the basic interval, over which the solution is defined, vary and replace the boundary value problem by an initial value problem with the width of the interval as independent variable. For analytical, as well as computational reasons, the initial value problem that ensues is more convenient. However, even linear boundary value problems lead to corresponding nonlinear initial value problems which have the form of Riccati equations.

The idea described above frequently has a clear physical interpretation and the quantity that satisfies the initial value problem has physical significance; for example, it may be the reflection coefficient in transport or wave processes. It was first used by Stokes (1862) in a somewhat crude discrete