Constructive methods for elliptic equations, by R. P. Gilbert, Springer Lecture Notes, no. 365, 1974, vii+397 pp.

In the last decades there has been a veritable explosion of research in partial differential equations. Much of the work involves applications of techniques and results of functional analysis, both linear functional analysis and topological results such as fixed point theorems. Somewhat apart from this main stream of investigations, there has been vigorous development in a direction which may be thought of as originating with Beltrami's representation of symmetric potentials in the last century and E. T. Whittaker's representation of solutions of Laplace's equation in three dimensions early in the present century. Bergman's integral operators, Vekua's generalised analytic functions, the present author's early work on singularities of harmonic functions are typical of this direction. Some of these developments were described in Gilbert's earlier book *Function theoretic methods in partial differential equations*, Academic Press, 1969.

The present work continues the account. It is based on lectures given by the author at the University of Delaware. Primarily these lectures were intended as a report on recent research by the group led by Gilbert at Indiana University and accordingly, they include much material not hitherto available in book form, and some material not at the time available in any printed form. Some discussion of the relation of this research to other investigations was also included, thus increasing the usefulness of these lecture notes.

A linear differential operator of the second order in two independent variables whose principal part is the Laplacian can be reduced to the form

$$L=\frac{\partial^2}{\partial z\,\partial\xi}+D(z,\xi)\frac{\partial}{\partial\xi}+F(z,\xi),$$

where $z=x+iy$ and $\xi=x-iy$ are regarded as independent complex variables. Solutions of $L[V]=0$ can be represented in the form

$$V(z,\xi)=\int_{-1}^{1}E(z,\xi,t)f(\tfrac{1}{2}z(1-t^2))(1-t^2)^{-1/2}\,dt.$$

There are other, related, forms involving the Riemann function associated with the (formally) hyperbolic operator L. A thorough study of E leads to an analysis of the correspondence between analytic functions f of a single complex variable and solutions of $L[V]=0$. In particular cases this approach can be utilized in boundary value problems involving the equations concerned, especially for the construction of approximate solutions (including numerical approximations). The method can be extended to elliptic equations of higher order and to a somewhat lesser extent to equations in more than two independent variables. It can be used for an analysis of the singularities of the solutions.

Function-theoretic methods can also be used for semilinear equations. This has been shown for certain second-order problems in the author's earlier book and is now extended to certain systems of equations of higher orders.

In the last two chapters weak (distributional) solutions of elliptic systems in two independent variables are discussed. The treatment of these establishes contact with earlier work by Bers (pseudo-analytic functions) and Douglis (hyperanalytic functions).

The investigations presented in this volume are highly technical and complex. It is probably inevitable, but from the reader's point of view far from welcome, that the presentation involves an unfavourable ratio of (often lengthy and involved) formulae to conceptual exposition. The series in which this volume appeared aims at rapid publication of topical material and is prepared to tolerate imperfection, but accepting this, one yet wishes that typing and proofreading had been more careful.

Notwithstanding these imperfections, this volume will be of considerable value to those engaged in research in related areas.

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The structure of fields, by David J. Winter, Graduate Texts in Mathematics, 16, Springer-Verlag, New York, 1974, xii+205 pp., $12.80.

David J. Winter's book, *The structure of fields*, is written in the form of a graduate level text book. The preface gives a glowing statement of objectives:

> This book is written with the objective of exposing the reader to a thorough treatment of the classical theory of fields and classical Galois theory, to more modern approaches to the theory of fields and to one approach to a current problem in the theory of fields, the problem of determining the structure of radical field extensions.

This statement is very misleading. It is true that there are chapters on basic algebra and group actions, elementary field theory, the structure of algebraic extensions, classical Galois theory, algebraic function fields and Galois theories involving algebraic structures other than groups, and a series of appendices supplementing these, but the presentation seems completely unsuitable for a beginning graduate student, or even for a more advanced student's first contact with field theory. Winter indicates that much of the material is based on courses he has taught on bialgebras and field theory. For others reaching either of these courses, Winter's book might be used as a supplementary reference, of value particularly for its exercises and its discussion of the bialgebra Galois theory of purely inseparable extensions of a field. However, I would not recommend it as the main text. For an advanced course on field theory, Jacobson, *Lectures in abstract algebra*. Vol. 3: *Theory of fields and Galois theory*, Van Nostrand, Princeton, N.J., 1964, covers practically all of the material in five out of the six chapters of Winter's book, and also includes several important topics not mentioned by Winter. Not only is his coverage more thorough, but Jacobson also makes significantly more of an effort to consider the student's viewpoint than