BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 5, September 1975

A GEOMETRIC PROOF OF THE STRONG MAXIMAL THEOREM

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Communicated by Robert Bartle, May 25, 1975

In R^n, suppose we consider the operator M_s given by

$$M_s(f)(x) = \sup_R \frac{1}{|R|} \int_R |f(y)|\, dy$$

where f is a locally integrable function on R^n and the sup is taken over all rectangles with sides parallel to the axes which contain the point x. Then the strong maximal theorem may be taken as the statement that M_s is bounded from $L(\log^+ L + 1)^{n-1}(Q)$ to weak $L^1(Q)$, i.e.

$$m\{M_s f > \alpha\} \leqslant A_n \int \frac{|f(x)|}{\alpha} \log^{n-1}\left(\frac{|f(x)|}{\alpha} + 1\right) dx$$

where A_n is some absolute constant, and Q is the unit cube in R^n.

Our result consists of a purely geometric argument establishing such an inequality. At the heart of the matter is a geometric proof of the following covering lemma:

Suppose $R_1, R_2, \ldots, R_k, \ldots$ is a sequence of rectangles contained inside the unit cube in R^n. Then there is a subcollection $\widetilde{R}_1, \widetilde{R}_2, \ldots$ of the R_k's satisfying the following conditions:

(1) $|\bigcup \widetilde{R}_k| \geqslant c_n |\bigcup R_k|$ for some absolute constant $c_n > 0$, and

(2) $\|\exp(\Sigma\, \chi_{\widetilde{R}_k})^{1/(n-1)}\|_{L^1} \leqslant C_n |\bigcup R_k|$ for some absolute constant $C_n < \infty$.

These observations lead to further results in the theory of differentiation of the integral.

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AMS (MOS) subject classifications (1970). Primary 42A68, 42A92; Secondary 42A18.