What kind of answers did I get to the five questions I posed at the beginning of this review? Rudin and Berberian both seem handy as references (I have used both in the last three months in my own research). Packel is not designed for this purpose at all, but is intended as a one semester course for advanced undergraduates. I could see it filling that role well if the class were good. Packel manages to pack many ideas into a small space by relegating the "general cases" of some theorems to the comment section at the end of each chapter and omitting others entirely. I fully support this procedure for the purpose intended. I believe that many mathematicians working in other fields could profitably "read through" Packel and get some useful insight into the methods and results of functional analysis.

Generally speaking the proofs are clear in each of the books. Packel makes his proofs clear for bright undergraduates, Rudin's are clear only for those who are trained to the Rudin style. His style is to leave items for the reader to fill in without saying he is doing so. The missing steps are no problem to someone who knows the subject well and is just looking around for the best way to prove a particular result. Rudin usually has the best way, and he often gets a bit more generality in the bargain. I would not recommend this Rudin book for self-study, but I plan to use parts of it in a graduate course next year. It is packed so full of nice theorems that anyone teaching a graduate analysis course can find something to use.

Berberian's style is informal; the text is laced with comments and historical notes (I prefer this to the "appendix for notes" system used by Rudin). His proofs are detailed enough for his audience. He is the best of the three when it comes to explaining what he is doing and where he is going. Packel is a close second while Rudin usually proceeds right to the business of writing beautiful proofs. Rudin also features four chapters on distribution theory with emphasis on Fourier transforms. While all Rudin's presentations are slick, this represents a big jump in that direction compared to other treatments I have seen.

In summary, the book-lover interested in functional analysis could do well to buy all three books for his bookshelf. For a quick course try Packel. For a year course select Berberian (if you lean toward operator theory) or Rudin (if you lean toward harmonic analysis).

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Random sets and integral geometry, by G. Matheron, Wiley, New York, 1975, xxiii+261 pp., $18.95

Geometry, according to Klein, is concerned with the action of a group G on a set S, and in particular with those properties of configurations in S which are invariant under G. Now it is a common phenomenon that, when G and S have some topological structure, there is an essentially unique measure μ on S which is invariant under G, and integral geometry is about the integration of functions f on S with respect to μ.

In a typical problem, the elements of S are geometrical objects (affine subspaces, spheres, finite sets of points) in Euclidean space R^d, and G is induced on S by a group of linear transformations of R^d. In such a situation the measure μ is usually easy to describe, but methods of great ingenuity are needed, and have been devised, to evaluate $\int f\,d\mu$ for particular functions f. The systematic development of this classical theory owes much to the work of Blaschke and his school (see **[1]** and **[10]**), but perhaps the most surprising results are those of Crofton, whose article **[4]** in the 1885 edition of the *Encyclopaedia Britannica* was far in advance of its time.

Much of the early work in integral geometry was motivated by pure mathematical curiosity and, for example, by the fact that integrals with respect to μ could be made to yield large families of affine invariants of convex bodies. But it was soon realised that there are practical problems to which the theory is relevant, in which μ describes a 'uniform distribution' over S. To be more precise, if μ has finite total mass, it can be normalised so that $\mu(S)=1$, and μ is then a probability measure invariant under G. Thus we can speak of a random geometrical object X, uniformly distributed over S in the sense that gX has the same distribution as X for all g in G. In this interpretation $\int f\,d\mu$ is just the expectation of the random quantity $f(X)$.

The first example of this situation is that of Buffon's needle (which is thrown on a floor ruled with parallel and equally spaced lines, and for which the probability of crossing a line is required **[2]**), but there are many others, and some of great practical importance. A useful survey may be found in **[6]**. Despite the many special techniques that are now known, some rather simple problems remain unsolved. For instance, Klee **[8]** has asked for the expected volume of the tetrahedron formed by four points chosen at random inside a tetrahedron of unit volume. The answer is an absolute constant whose exact value is still unknown. (It is known if the tetrahedron is replaced by a sphere, as is that of the corresponding problem in R^d for any d **[7]**.)

The measure μ is not, however, always totally finite. In the simplest case, where S is R^d itself and G is the Euclidean group, μ is of course Lebesgue measure and $\mu(R^d)=\infty$. Then the natural replacement of the single random object X is an infinite collection of such objects forming a 'Poisson process'. This idea applies to quite general S; a Poisson process on S is a random subset Ξ of S, such that every subset A of S with $\mu(A)<\infty$ contains finitely many elements of Ξ, the probability that A contains exactly n elements being $e^{-\mu(A)}\mu(A)^n/n!$, and the numbers in disjoint sets A being independent random variables.

Such Poisson processes have both theoretical and practical importance. For example, if S is the collection of lines in R^2, the corresponding process has been used as a model for the fibres making up a sheet of paper. A typical function of Ξ might be the area of the polygon containing the origin and bounded by the lines of Ξ, and such functions can be integrated by techniques which are now well understood (largely because of the work of Miles **[9]**). But of course the Poisson process is only one model for a random subset of S, and it may well not fit the facts of a particular application. For this reason there has come in recent years to be more interest in a general theory of random sets.

If Ξ is a random subset of S, the first problem is to describe the stochastic structure of Ξ in a compact and convenient way. The usual way of doing this for the Poisson and related processes has been through the family of random variables $N(A)$, where A runs over a small class of sets chosen so that $N(A)$, the cardinality of $\Xi \cap A$, is finite. It is simple to write down the conditions that the joint distributions of this family must satisfy, though technical difficulties arise in reversing the process to construct a random set Ξ having a given consistent collection of joint distributions.

The decisive objection to the specification by means of $N(\cdot)$, however, is that there are interesting random sets which are sufficiently substantial that $N(A)=\infty$ with positive probability for the convenient sets A. For this reason Kendall (following Davidson) developed in [**5**] a means of specifying Ξ by just noting whether or not Ξ meets a variable set A. It turns out that, if a reasonably rich family of sets A is considered, then the stochastic structure of Ξ is determined by the set function $H(\cdot)$, where $H(A)$ is the probability that $\Xi \cap A$ is nonempty. Thus, for instance, the Poisson process and that alone has $H(A)=1-e^{-\mu(A)}$. As a set function, H is an 'alternating capacity of infinite order' in the sense of Choquet [**3**], and Choquet's results imply that, under topological conditions, every such capacity (suitable normalised) comes from a random set.

This result, which is proved under broader and more natural conditions in [**5**], is the starting point of Matheron's book. He sets out to explain Kendall's analysis to an audience raised on Bourbaki, but alas without Bourbaki's penetrating insight into the essential structure of a particular mathematical area. Thus the book begins with an elaborate account of various topologies for classes of subsets of R^d, with the aim of deriving the fundamental existence theorem from Choquet's results. The resulting subordination of probabilistic to topological ideas obscures the simplicity of the basic structure, and is bound to make the book less accessible to the nonspecialist.

For it is clear that the author's intention is not just to expound an elegant piece of pure mathematics, but to provide a tool for application to models of random sets in the real world. For this he is well served by his experience in morphology and stereology, and indeed when the Bourbaki uniform is stripped away, there stand revealed some very valuable ideas in this direction. The stress on convexity (Ξ is convex if and only if H satisfies a certain additivity condition) and on the precise definitions of granulometry and size distribution, could well prove influential. But a weakness is the excessive reliance on the single function H; in applications there are other functions of Ξ (like $\mu(\Xi \cap A)$ as a function of A) which are sometimes likely to be more tractable.

It is good that a book has been written on these subjects. This book has been written by an author with a deep knowledge of their theoretical and applied aspects, and it is one which the specialist will find valuable and stimulating. It would have been possible to write a more readable, a better balanced, and a more widely appealing book, and it is a shame that the opportunity has been missed.

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Monotone matrix functions and analytic continuation, by W. F. Donoghue, Jr., Springer-Verlag, New York, Heidelberg, Berlin, 1974, 182 pp., $19.70

In a 1934 article Charles Loewner posed and solved the following problem: Characterize the class $P_n(a, b)$ of real-valued functions on the interval (a, b) that are *monotone matrix functions of order n*. This means that whenever A, B are n-by-n Hermitian matrices with spectrum in (a, b) and $A \geqq B$ (i.e. $A-B$ is positive definite), then $f(A) \geqq f(B)$. As usual, $f(A)$ is defined as the Hermitian matrix whose eigenvectors are the same as those of A and whose eigenvalues are gotten from those of A by applying f. Loewner showed that for $n \geqq 2$ such a function is automatically continuously differentiable and, regarded as a function from the linear space of n-by-n Hermitian matrices to itself, its derivative at $A = \operatorname{diag}(\lambda_1, \cdots, \lambda_n)$ sends the matrix (X_{jk}) to the matrix $([\lambda_j, \lambda_k]_f X_{jk})$, where

$$[x, y]_f = \begin{cases} \dfrac{f(x)-f(y)}{x-y} & \text{if } x \neq y, \\ f'(x) & \text{if } x = y. \end{cases}$$

So a necessary and sufficient condition for monotonicity of order n is the positive definiteness of the matrix $[\xi_j, \xi_k]_f$ for every choice of $\xi_1, \cdots, \xi_n \in (a, b)$. An equivalent condition is the positive definiteness of $[\xi_j, \eta_k]_f$ for every $a < \xi_1 < \eta_1 < \xi_2 < \cdots < \eta_n < b$; in fact Loewner starts with proving the necessity