arise when the symbol is pointwise invertible, the author presents an intensive study of what happens when the symbol is permitted to have certain kinds of zeros. Briefly, zeros are allowed which arise when the function can be factored into the product of some canonical function (usually a polynomial) with a nonvanishing continuous function. The operator defined by the canonical function is studied in considerable detail usually by putting an appropriate norm on the range relative to which the operator is well behaved. The study of the general case is then reduced to this.

A number of books have appeared recently on related topics. Perhaps the closest is the book of I. C. Gohberg and I. A. Fel′dman, *Convolution equations and the projection method for their solution*, "Nauka", Moscow, 1971; Amer. Math. Soc., Providence, R.I., 1974. Much of the material on "abstract" singular integral operators mentioned above also appears in this book including a treatment with considerable detail of the solution of such equations by the projection method. In the book of Prössdorf, the latter topic is discussed in a short appendix. Another related book is by I. C. Gohberg and N. J. Krupnick, *Introduction to the theory of one-dimensional singular integral operators*, "Shtintsa" Kishiev, 1973, which takes up the study of singular integral operators on contours in considerable detail with coefficients from a variety of spaces. Again there is considerable overlap, but operators with discontinuous coefficients are treated by Prössdorf only in an appendix. Lastly, there are two books by the reviewer, *Banach algebra techniques in operator theory*, Academic Press, New York, 1972, and *Banach algebra techniques in the theory of Toeplitz operators*, (CBMS Regional Conference Series, No. 15) Amer. Math. Soc., Providence, 1973, which confine attention to the study of Toeplitz operators on Hilbert space with all kinds of coefficients.

The book under review presents a modern unified treatment of these classes of operators. It is readable and complete on the topics it emphasizes. Its chief virtues are the material on operators not of normal type, a treatment of Fredholm theory in Fréchet spaces, and a lengthy bibliography of the area, especially of the Soviet literature which is sometimes difficult to obtain.

R. G. DOUGLAS

Lectures in functional analysis and operator theory, by Sterling K. Berberian, Springer-Verlag, New York, 1974, ix+345 pp., $14.80

Functional analysis, a short course, by Edward W. Packel, Intext Educational Publishers, New York, 1974, xvii+172 pp., $10.00

Functional analysis, by Walter Rudin, McGraw-Hill, New York, 1973, xiii+397 pp.

I liked these three books and enjoyed reviewing them. My viewpoint when reading these books was most definitely not that of a student. I did not read all (or even most) proofs in detail, nor did I carefully check formula references and the like. My main interests were:

(1) How is the material organized?

(2) Are the proofs clear?

(3) Is there sufficient explanatory material to make clear the author's direction and to describe the historical reasons for the development?

(4) How would the book be used in a course?

(5) Would I keep the book handy as a reference?

Before I turn to a discussion of these points let me briefly describe the nature of this subject called functional analysis. Here is the first paragraph of the Preface to Berberian's book:

> An analyst is a mathematician who is habitually seen in the company of the real or complex numbers; a "functional analyst" is an analyst who is not squeamish about using Zorn's lemma, definitely relishes the use of topology and does not stand in the way of the internal algebraic impulses of the subject.

This definition would seem to be shared by Packel and Rudin as well, but I admit to a bit of squeamishness about Zorn's lemma after reading Solovay (Ann. of Math. **92** (1970), pp. 1–56) and Garnir (*Functional analysis and its applications*, Springer-Verlag, Berlin, 1974, pp. 189–204). These papers serve to drive home the fact that the Axiom of Choice (in its strongest form) is only one of the possibilities (and perhaps is not always the best one).

The basic object of interest in functional analysis is the topological vector space. All three books get into this material promptly (although Berberian does some material in the more general setting of topological groups). The standard examples of such spaces are Hilbert spaces, L^p spaces, and continuous function spaces. These spaces are analyzed in each book. Finally, each book presents the theory of Banach algebras as an example of the best mixture of algebra and analysis.

The most pleasing theorems in Banach algebra theory are those which link the algebraic structure and the analytical structure (e.g. the multiplication and the norm). The key element in such theorems is always the completeness of the algebra in its norm. This is generally used via the Baire category theorem or by asserting that a given series converges to an element of the algebra. The spectral radius formula, which is contained in all three books, is a beautiful example of a theorem of this type. For this theorem let x be any element of the Banach algebra A and recall that a complex number λ lies in the spectrum of x if the element $(x-\lambda 1)$ has no inverse in A. (Here we assume A has a unit element 1 for simplicity.) Then

$$\lim_{n\to\infty} \|x^n\|^{1/n} = \text{lub}\{|\lambda| : \lambda \text{ is in the spectrum of } x\}.$$

Notice that the left side depends only on the norm and *not* on the specific Banach algebra A in which x lies (since any algebra containing x will contain all x^n). However, the right side would seem to depend strongly on the specific form of the Banach algebra A since the spectrum would tend to become smaller in a larger algebra.

What kind of answers did I get to the five questions I posed at the beginning of this review? Rudin and Berberian both seem handy as references (I have used both in the last three months in my own research). Packel is not designed for this purpose at all, but is intended as a one semester course for advanced undergraduates. I could see it filling that role well if the class were good. Packel manages to pack many ideas into a small space by relegating the "general cases" of some theorems to the comment section at the end of each chapter and omitting others entirely. I fully support this procedure for the purpose intended. I believe that many mathematicians working in other fields could profitably "read through" Packel and get some useful insight into the methods and results of functional analysis.

Generally speaking the proofs are clear in each of the books. Packel makes his proofs clear for bright undergraduates, Rudin's are clear only for those who are trained to the Rudin style. His style is to leave items for the reader to fill in without saying he is doing so. The missing steps are no problem to someone who knows the subject well and is just looking around for the best way to prove a particular result. Rudin usually has the best way, and he often gets a bit more generality in the bargain. I would not recommend this Rudin book for self-study, but I plan to use parts of it in a graduate course next year. It is packed so full of nice theorems that anyone teaching a graduate analysis course can find something to use.

Berberian's style is informal; the text is laced with comments and historical notes (I prefer this to the "appendix for notes" system used by Rudin). His proofs are detailed enough for his audience. He is the best of the three when it comes to explaining what he is doing and where he is going. Packel is a close second while Rudin usually proceeds right to the business of writing beautiful proofs. Rudin also features four chapters on distribution theory with emphasis on Fourier transforms. While all Rudin's presentations are slick, this represents a big jump in that direction compared to other treatments I have seen.

In summary, the book-lover interested in functional analysis could do well to buy all three books for his bookshelf. For a quick course try Packel. For a year course select Berberian (if you lean toward operator theory) or Rudin (if you lean toward harmonic analysis).

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Random sets and integral geometry, by G. Matheron, Wiley, New York, 1975, xxiii+261 pp., $18.95

Geometry, according to Klein, is concerned with the action of a group G on a set S, and in particular with those properties of configurations in S which are invariant under G. Now it is a common phenomenon that, when G and S have some topological structure, there is an essentially unique measure μ on S which is invariant under G, and integral geometry is about the integration of functions f on S with respect to μ.