BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 6, November 1974

SOME EXTENSION THEOREMS FOR REGULAR MAPS OF STEIN MANIFOLDS

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Communicated April 22, 1974

The central result of this paper is an analogue, in the category of complex manifolds and holomorphic maps, of the tubular neighborhood theorem. The following theorems are proved.

THEOREM A. *Let* S *be a Stein manifold and let* $f\colon S \longrightarrow M$ *be a holomorphic embedding. Let* $K \subset S$ *be compact and let* N_f *be the normal bundle of* f. *We identify* S *with the zero section of* N_f. *Then there is a neighborhood* U *of* K *in* N_f *and a holomorphic embedding* $F\colon U \longrightarrow M$ *such that* $F | U \cap S = f$.

THEOREM B. *The word "embedding" can be replaced by the word "immersion" in the above theorem.*

THEOREM C. *Let* $f\colon S \longrightarrow M$ *be a holomorphic map, where* S *is Stein, such that* f *is regular at* $x_0 \in S$. *Let* $K \subset S$ *be compact with* $x_0 \in K$. *Then there is a trivial bundle* $\widetilde{A}$ *over* S, $\dim_{\mathbf{C}} \widetilde{A} = \dim_{\mathbf{C}} M$, *a neighborhood* U *of* K *in* $\widetilde{A}$, *and a holomorphic map* $F\colon U \longrightarrow M$ *such that* $F | U \cap S = f$ *and* F *is regular at* x_0 *(again* S *is identified with the zero section of* $\widetilde{A}$*).*

When M above is itself a Stein manifold, Theorems A and B are known and were proved by Forster and Ramspott. So the main effort of our work is to construct a strictly plurisubharmonic function ϕ on a neighborhood of $f(K)$ (in Theorem A) such that $\phi^{-1}((-\infty, b])$ is compact for all $b \in R$. Theorem B follows then from Theorem A by a well-known result. Theorem C follows from Theorem A by embedding a neighborhood of $g(K)$ in $S \times M$ in some $\mathbf{C}^q$ $(g(x) = (x, f(x)))$ and using the holomorphic retraction theorem and other standard results.

A special case of Theorem A, namely the case where S is a disc, can

AMS (MOS) subject classifications (1970). Primary 32H99, 32C10.

be applied to show that the differential metric associated to the Kobayashi metric on a complex manifold is upper semicontinuous.

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