BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 4, July 1974

STUNTED PROJECTIVE SPACES AND THE J-ORDER OF THE HOPF BUNDLE

BY S. FEDER AND S. GITLER

Communicated by William Browder, December 17, 1973

Let FP^n be the projective space of dimension n over F where $F=C$, Q (Q stands for quaternions). We have the natural cofibrations

$$FP^{m-1} \to FP^n \to FP_m^n$$

for $m \leqq n$.

Let H denote the Hopf bundle over FP^n. It is well known that the stunted projective space FP_k^{n+k} can be identified with the Thom space of kH over FP^n, this last being denoted by $(FP^n)^{kH}$.

We study the stable homotopy types of stunted projective spaces FP_k^{n+k} for $F=C$ and $F=Q$ and obtain a complete classification. This classification is given in terms of the J-order of the Hopf bundle over FP^n.

We denote by A_n the J-order of H over CP^n, and by B_n the J-order of H over QP^n. With this notation the results are:

THEOREM A. *The spaces $(CP^n)^{kH}$ and $(CP^n)^{lH}$ are of the same stable homotopy type if and only if one of the following conditions holds*: $(n\neq 2, 4)$[1]

(i) $k-l\equiv O(A_n)$,

(ii) $k-l\equiv O(A_{n-1})$ *and* $k+l\equiv O(A_n)$,

(iii) $k-l\equiv O(A_{n-1})$ *and* $k+l+2(n+1)\equiv O(A_n)$.

THEOREM B. *The spaces $(QP^n)^{kH}$ and $(QP^n)^{lH}$ are of the same stable homotopy type if and only if one of the following conditions holds*:

(i) $k-l\equiv O(B_n)$,

(ii) $k-l\equiv O(B_{n-1})$ *and* $k+l\equiv O(B_n)$.

In [1] we have proven that the conditions in Theorem A are necessary and observed that $A_n=A_{n-1}$ for n odd which completed the classification

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AMS (MOS) subject classifications (1970). Primary 55D15, 55G25; Secondary 55F50.

[1] The cases $n=2, 4$ are also solved but the numerical conditions will not be stated here.

for that case. To complete the classification we construct the necessary homotopy equivalences. We also prove[2]

THEOREM C. *The J-order B_n of the Hopf bundle over QP^n is given by*

$$\nu_2(B_n) = \max\{2n + 1, 2j + \nu_2(j) \mid 1 \leqq j \leqq n\}$$

and

$$\nu_p(B_n) = \max\{ j + \nu_p(j) \mid 1 \leqq j \leqq 2n/(p - 1)\}$$

when p is an odd prime.

Here $\nu_p(r)$ is the highest exponent of the prime p which divides r.
The proofs will appear elsewhere.

REFERENCE

1. S. Feder and S. Gitler, *Stable homotopy types of stunted complex projective spaces*, Proc. Cambridge Philos. Soc. **73** (1973), 431.

CENTRO DE INVESTIGACION Y ESTUDIOS AVANZADOS DEL IPN, APARTADO POSTAL 14-740, MEXICO 14, D.F.

[2] This result was announced by F. Sigrist and U. Sutter, *Cross-sections of symplectic Stiefel manifolds*, Notices Amer. Math. Soc. **19** (1972), A-214, but no proof has been published.