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ON A CLASS OF MINIMAL CONES IN R^n

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1. **Introduction.** In what follows let $S^p(\rho)=\{x \in R^{p+1} \mid |x|=\rho\}$ and $S_{p,q}=S^p((p/(p+q))^{1/2}) \times S^q((q/(p+q))^{1/2}) \subset S^{p+q+1}(1)$. Let M be a codimension 1, closed minimal submanifold of $S^{n+1}(1)$ and $C(M)=\{tx \mid 0<t<1, x \in M\}$.

It is well known that $C(M)$ is a minimal submanifold of R^{n+2}. An important question is whether $C(M)$ minimizes area in R^{n+2} with respect to its boundary M. With respect to this question the following results are known:

(a) When $n \leqq 5$, Simons [4] has given a negative answer.

(b) When $M=S_{p,p}$, $p \geqq 3$, Bombieri-De Giorgi-Giusti [1] have given an affirmative answer.

(c) When $M=S_{p,q}$ and either $p+q \geqq 7$ or $p=q=3$ Lawson [2], using a different approach from Bombieri-De Giorgi-Giusti, has given an affirmative answer.

(d) Lawson has also proved that when $n=6$ or $n=7$ the set of minimal cones $C(M)$, that minimize area in R^{n+2} with respect to their boundary M, is finite up to diffeomorphisms.

In this note we answer the question when $M=S_{p,q}$ with $p+q=6$.

2. **Results.** Using techniques related to those of Bombieri-De Giorgi-Giusti, we were able to prove in [3] the following two theorems:

THEOREM 1. *If $p+q=n$ and either*
(a) $n \geqq 7$ *or*
(b) $n=6$ *with* $|p-q| \leqq 4$,
then the cone $C(S_{p,q})$ minimizes area in R^{n+2} with respect to its boundary $S_{p,q}$.

THEOREM 2. *$C(S_{1,5})$ and $C(S_{5,1})$ do not minimize area in R^8 with respect to their respective boundaries $S_{1,5}$ and $S_{5,1}$.*

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Now let V be a C^2 vector field in R^{n+2}, having compact support not containing $S^{n+1}(1)$ and let $\{\phi_t\}$ be its 1-parameter group of diffeomorphisms. We say that $C(M)$ is stable if for any such vector field there is $\varepsilon > 0$ such that

$$\text{Area of } \phi_t(C(M)) \geqq \text{Area of } C(M) \quad \text{when } |t| < \varepsilon.$$

By an argument similar to one in Simons [**4**] one may prove that $C(S_{1,5})$ and $C(S_{5,1})$ are stable.

So we have the following:

THEOREM 3. *Although $C(S_{1,5})$ and $C(S_{5,1})$ are stable they do not minimize area in R^8 with respect to their respective boundaries $S_{1,5}$ and $S_{5,1}$.*

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