with 1 (including characteristic 2). There are many exercises throughout the book, a bibliography and an index. Also there are a number of trivial misprints, none of which should cause the reader difficulty.

What this sketchy listing of some of the contents of this book fails to convey is the superb organization of the vast amount of material which the author has included. Representation theory is basic to the entire presentation. It is not only that individual topics are interesting in themselves as they are developed, but their impact is felt consistently throughout the remaining pages. This is a book which one can come back to again and again, gaining new insights every time.

R. D. SCHAFER

Introduction to the Arithmetic Theory of Automorphic Functions by Goro Shimura. Iwanami Shoten, Publishers, Tokyo; Princeton University Press, Princeton, New Jersey.

Shimura's book *Introduction to the Arithmetic Theory of Automorphic Functions* is one of the most significant additions to the literature on automorphic function theory in many years. In this remarkably slender volume, Professor Shimura manages to build up the theory of automorphic functions from scratch and at the same time present much recent work which was previously available only in the original papers.

Chapter 1 is devoted to a study of a discrete subgroup Γ of $SL(2, \mathbf{R})$ acting on the complex upper half-plane H by linear fractional transformations. If H^* denotes the union of H and the cusps of Γ, then it is shown that the quotient space H^*/Γ carries a natural structure of a Riemann surface. If the Riemann surface H^*/Γ is compact, then Γ is said to be a *Fuchsian group of the first kind*. The theory is illustrated by means of the elliptic modular group and its congruence subgroups, all Fuchsian groups of the first kind, and the genera of all these Riemann surfaces are computed.

Chapter 2 proceeds to study automorphic functions and automorphic forms with respect to a Fuchsian group of the first kind. The basic facts concerning compact Riemann surfaces are assumed. From these, the dimension of the spaces of holomorphic automorphic forms and cusp forms of given weight with respect to Γ are computed. Moreover, the volume of a fundamental domain for Γ is computed.

The arithmetic theory begins in Chapter 3 which is devoted to the theory of Hecke operators. Shimura develops the theory of Hecke operators axiomatically. At first, the Hecke operators appear as elements of an abstract ring, and only later do the classical Hecke operators arise from representations of the abstract ring on the spaces of automorphic forms of various weights. The end result of the chapter is Hecke's theory of zeta functions corresponding to automorphic forms.

In Chapter 4, the level of difficulty increases dramatically. The subject is the theory of elliptic curves and the basic facts are stated without proof. Deductions from the basic facts, however, are proved. It is unfortunate that more details are not provided in this chapter, since the results quoted are not always accessible to nonexperts. The reader is referred to an Appendix for the statements of the main tools required from algebraic geometry.

Chapter 5 fulfills one of the principal aims of the book—the treatment of the theory of complex multiplication of elliptic curves and its application to the construction of class fields over an imaginary quadratic field. The problem treated here is to generate the class fields of a given algebraic number field F by means of special values of one or more analytic functions. In case $F = \boldsymbol{Q}$, the solution is given by the theorem of Kronecker-Weber which asserts that every abelian extension of $\boldsymbol{Q}$ is contained in a cyclotomic field $\boldsymbol{Q}(e^{2\pi i/n})$. Thus, special values of the exponential function suffice to generate the class fields. If $F =$ an imaginary quadratic field, the class fields can be obtained by adjoining to F special values of the elliptic modular invariant $j(\tau)$ and the Weierstrass $\wp$-function and its derivative. This is the main result of Chapter 5. The case of general F is still open and is Hilbert's 12th problem. However, what is known today concerning this problem is due to the beautiful work of Shimura and he gives some indication of what is known in the general case at the end of Chapter 5.

Chapter 6 is devoted to a further exploration of the theory of complex multiplication of elliptic curves and its relation to the field of all modular functions of arbitrary level whose Fourier coefficients belong to a cyclotomic field.

In Chapter 7 a second major theme of the book is taken up—the zeta functions of algebraic curves and abelian varieties. Here the major outstanding problem is the conjecture of Hasse and Weil concerning the analytic continuability of the zeta function of an algebraic variety defined over an algebraic number field. In Chapter 7, this conjecture is verified in a number of cases. It turns out that the zeta functions of certain of the varieties investigated are intimately tied up with the theory of class fields over real quadratic fields. This connection has been the subject of a recent series of papers by Shimura.

Chapter 8 is devoted to an exposition of the so-called Eichler cohomology associated to a Fuchsian group.

Finally, Chapter 9 is a brief discussion of some Fuchsian groups which are not obtained as groups of congruence type, namely Fuchsian groups obtained from quaternion algebras.

Chapters 4–9 are formidable to the novice, but this is, to some extent, understandable, since they are built on an incredibly broad foundation, including the theory of abelian varieties and class field theory. Moreover, they reflect the imperfect state of a theory which is still in the process of evolving. But this is not the fault of the author. Indeed, the latter chapters of the book point to the extremely broad and profound influence Shimura's work has had on the theory of automorphic functions. The mathematical community should be grateful to Professor Shimura for providing this book to present the current status of the field as well as to stimulate others to work on the many avenues of investigation which his work suggests.

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