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TOTALLY GEODESIC FIBRE MAPS[1]

BY MU-CHOU LIU

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Let M be a Riemannian manifold and $\Pi: TM \to M$ be its tangent bundle. There exist two kinds of naturally induced metrics on TM, the Sasaki metric and the pseudo-Riemannian metric ([**3**], [**4**]). If TM is endowed with the Sasaki metric and M is compact, we have shown that TM is a complete Riemannian manifold which admits no negative curvature. In [**4**], Yano and Kobayashi determined the holonomy group of the pseudo-Riemannian connection on TM. A fibre map is said to be trivial if it collapses the whole fibre into a point.

Based on the results of Yano and Kobayashi, we prove the following

THEOREM 1. *Suppose M and N are Riemannian manifolds. If $F: TM \to TN$ is a totally geodesic fibre preserving map, then the induced map $f: M \to N$ is totally geodesic. If for some $u \in TM$,* $\operatorname{Ker} F_{*_u}$ *contains a nonvertical vector, then F is trivial.*

By using the Morse theory and Cartan-Hadamard Theorem together with the above theorem, we prove the following

THEOREM 2. *Suppose M is a Riemannian manifold, and suppose its Ricci curvature K satisfies $K(X, X) \geqq (n-1)/c^2$ for every unit vector X at every point of M, where c is a positive constant. If there exists a geodesic of length greater than Πc, and if N is a complete Riemannian manifold of negative curvature, then any fibre preserving totally geodesic map $F: TM \to TN$ is trivial.*

COROLLARY. *If $f: M \to N$ is a map such that the tangent map $f_*: TM \to TN$ is totally geodesic, then f is a constant map.*

A direct consequence of Theorem 2 is the following:

THEOREM 3. *Suppose M is a compact Riemannian manifold with everywhere positive definite Ricci tensor. If N is a Riemannian manifold of negative curvature, then any fibre preserving totally geodesic map $F: TM \to TN$ is trivial.*

The proofs of these results will appear in [**2**].

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Department of Mathematics, University of Illinois at Chicago Circle, Chicago, Illinois 60680