BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 3, May 1972

RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited. Manuscripts more than eight typewritten double spaced pages long will not be considered as acceptable. All research announcements are communicated by members of the Council of the American Mathematical Society. An author should send his paper directly to a Council member for consideration as a research announcement. A list of the members of the Council for 1972 is given at the end of this issue.

THE FIRST BETTI NUMBERS OF CERTAIN LOCALLY TRIVIAL FIBRE SPACES

BY BILL WATSON

Communicated by Emery Thomas, November 15, 1971

It is well known that, for a compact, oriented, homogenous coset space G/H arising from a compact, semisimple Lie group G, one has

$$b_1(G/H) \leqq b_1(G) = 0.$$

In this note, we announce the following generalization of that result:

THEOREM 1. *Let $\pi : E \to B$ be a locally trivial Riemannian fibre space, E and B compact, oriented Riemannian manifolds, with the fibres $F = \pi^{-1}(b)$ immersed in E as minimal submanifolds. Then*

$$b_1(B) \leqq b_1(E).$$

We outline the proof.

From Hodge-deRham theory, we have $H^p(M, \boldsymbol{R}) \cong \mathscr{H}^p(M)$, the space of harmonic p-forms on the compact, oriented Riemannian manifold M. In [**4**], we show

THEOREM 2. *Fix $p \geqq 1$. If $\varphi : E \to B$ is a locally trivial fibre space mapping between compact, orientable Riemannian manifolds satisfying $\varphi^*\delta = \delta\varphi^*$ on all p-forms of the base manifold, then*

$$b_p(B) \leqq b_p(E).$$

We also show

THEOREM 3. *Fix $p \geqq 1$. Then $\varphi : E \to B$, a C^3 map between arbitrary compact oriented Riemannian manifolds, commutes with the codifferential*

AMS 1970 *subject classifications.* Primary 55F05.
Key words and phrases, Netti numbers, fibre spaces, Hodge theorem, harmonic mappings, minimal submanifolds.

Copyright © American Mathematical Society 1972

operator, δ, on p-forms of B if and only if

(a) *φ is a C^∞ locally trivial Riemannian fibre space mapping, and*

(b) $\tilde{\delta}(\varphi_* \wedge \cdots \wedge \varphi_*)$*(p-times)* $= 0$,

where $\varphi_* \wedge \cdots \wedge \varphi_*$ is the canonical tensor-valued p-form of type $(p, 0)$ defined in [**2**] *and* [**4**] *and $\tilde{\delta}$ is the codifferential operator for such tensor-valued forms, dual to the exterior differentiation operator $\tilde{d}$ defined in* [**4**].

It is known that φ is a harmonic mapping [**1**] if and only if $\tilde{\Delta}(\varphi_*) = 0$ if and only if $\tilde{\delta}(\varphi_*) = 0$. Moreover, a locally trivial fibre space mapping is harmonic if and only if its fibres are minimally immersed, because $\tilde{\delta}\varphi_*$ is essentially the trace of the O'Neill T-tensor for the fibre space mapping φ and T is the second fundamental form of the fibres when restricted to vertical vectors. Hence, Theorem 1 is outlined. Full proofs will appear elsewhere [**4**].

There are several immediate corollaries to Theorem 1. For instance,

COROLLARY 1. *Let $\pi : P \to M$ be a Riemannian principal fibre bundle with compact Lie structure group G, and both P and M compact (e.g. bundle of frames of M). Then*

$$b_1(M) \leqq b_1(P).$$

Theorem 1 is a generalization of other similar results for the Laplacian operator found in [**3**].

REFERENCES

1. J. Eells, Jr. and J. H. Sampson, *Harmonic mappings of Riemannian manifolds*, Amer. J. Math. **86** (1964), 109–160. MR **29** #1603.

2. A. Lichnerowicz, *Applications harmoniques et variétés Kähleriennes*, Sympos. Math. (INDAM, Rome, 1968/69), vol. 3, Academic Press, London, 1970, pp. 341–402. MR **41** #7598.

3. B. Watson, *Manifold maps which commute with the Laplacian on p-forms* (submitted for publication).

4. ———, *δ-commuting maps and Betti numbers* (submitted for publication).

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF WASHINGTON, SEATTLE, WASHINGTON 98105