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BOOK REVIEWS

Modern Applied Algebra by Garrett Birkhoff and Thomas C. Bartee. McGraw-Hill, New York, 1970. 431 pp.

Over the past forty years or so, modern (abstract) algebra, as envisaged by van der Waerden in his classic book, has become a well-accepted, standard course topic in most college mathematics curricula. This corresponds, of course, to the increasing importance of algebraic thought in many branches of theoretical mathematics. The past two or three decades have now brought a surprising growth in the applications of abstract algebraic concepts and results in various outside areas. The best-known —but by no means only—examples of this are probably applications in electronic engineering and computer science, such as the uses of Boolean algebra in connection with switching networks, the development of algebraic coding theory, and, more recently, the algebraic study of finite state machines and of formal languages.

Very few of our college mathematics departments have taken much notice of these applications of abstract algebra, and special courses on the particular topics mentioned are now most often found in electrical engineering or computer science departments. However, the last years have seen a developing awareness of the need for the mathematics community to accept a responsibility for broader educational opportunities in a more encompassing "mathematical science" in which students may explore the areas of overlap between mathematics, its applications, and scientific computing.

The present book is an important contribution to this need for a broadening of mathematical education. Its aim is to present a sound introduction to basic ideas and techniques of modern algebra which have proved to be useful in certain applications while, at the same time, familiarizing the reader with these applications themselves. It addresses itself first and foremost to mathematics students with interests in scientific computing although it could be very useful as well for students from, say, computer science or electrical engineering. At the same time, the work does not really constitute an applied mathematics text in the narrow sense since its emphasis is predominantly on theorem proving rather than on problem solving.

By necessity the authors had to concentrate on certain specific applications of algebra, and they chose for these some of the problem areas cited above related to data communication and the design of switching networks; in particular, a recurring problem considered throughout the text is that of optimal coding of binary information. Correspondingly, the principal algebraic structures discussed are Boolean algebras, monoids and groups, lattices, rings and finite fields. A better impression of the range of topics presented may be obtained from a brief summary of the content of the book.

Chapters 1 and 2 constitute an introduction to various fundamental concepts, such as the notions of sets, functions, binary relations and graphs, together with a first informal discussion of algebraic systems, such as Boolean algebras on power sets, the monoid of functions from a set into itself, the Peano axioms for the positive integers, cyclic unary algebras, etc. This is followed in Chapter 3 by an introduction to finite state machines as a mathematical characterization of the process of computing. The covering and equivalence relations of such machines are discussed as well as procedures for minimizing the number of states. Chapter 4 fits only peripherally into the algebraic framework; it presents the elements of the algorithmic language ALGOL.

With Chapter 5 then begins the axiomatic treatment of algebraic structures. Boolean algebras are defined and discussed formally and then in Chapter 6 applied to logic design and the optimization problem in switching network design. Chapter 7 is devoted to the elements of the theory of monoids and groups up to Lagrange's theorem and the definition of normal subgroups. As an application, this is followed in Chapter 8 by a discussion of binary group codes.

Chapter 9 turns to partial orderings and lattices, and ends with a proof of the finite case of Stone's representation theorem for Boolean algebras. Then Chapter 10 concerns the elements of the theory of rings and ideals up to and including the unique factorization theorem, followed in Chapter 11 by a study of polynomial rings and their application to the construction and analysis of polynomial codes. Fields are first introduced in Chapter 10; thereafter Chapter 12 addresses itself to finite fields and their application to Bose-Chaudhuri-Hocquenghem codes. Chapter 13 is entitled "Recurrent Sequences" and presents an introduction to linear difference equations, formal power series, generating functions, and the study of difference codes. The final Chapter 14 again transcends the algebraic framework and discusses the concept of computability in relation to ideas from automata theory, mathematical linguistics, and programming languages.

In any book of this type, there is inevitably some separation between theoretically oriented sections and chapters and those devoted to applications. In general, the authors have clearly succeeded in minimizing this separation, although a few instances appear to reflect some lack of correlation between theoretical results and their practical uses. In line with the overall aim of the book, the presentation in the theoretically oriented parts—in format and precision—is that of a modern mathematics text. At the same time, a clear effort was made in the applications oriented parts to avoid assumption of specialized prerequisite knowledge of electrical engineering or computer science. As a result, some of the applied parts seem to be comparatively more elementary and discursive than the theoretical ones.

As with any text, there will certainly be some disagreement with the authors' choice of topics. This may not so much pertain to the algebraic material as to the selection of the applications. For instance, the stress on algebraic coding theory may be questioned by some readers with interests in computer science who might want to see further discussions on, for example, the algebraic theory of automata and formal languages instead of some material on certain classes of codes. A more objective criticism might relate to the chapter on ALGOL, which is somewhat weaker than the other chapters in the book. One reason for its inclusion was probably the desire to provide a foundation for the ALGOL algorithms in later chapters and to discuss some basic aspects of formal languages. Surprisingly, ALGOL procedures are never introduced, and later on only the Gaussian elimination algorithm is written in the form of a procedure.

These comments cannot in any way detract from the considerable value of the book as a text for various courses of a new and urgently needed type. In its entirety the material can be taught as a year course on the advanced undergraduate/beginning graduate level, and the preface indicates that at Harvard University it is indeed so taught. It should also be possible to select appropriate topics for meaningful semester courses. Numerous exercises have been included throughout the book which should enhance its value as a text even further.

All in all, this is a significant addition to the mathematics text market, which deserves widespread and very thoughtful attention and, hopefully, will stimulate in many institutions the introduction of courses following its ideas.

WERNER C. RHEINBOLDT

Lebesgue's Theory of Integration by Thomas Hawkins. University of Wisconsin Press, Madison, 1970. xv + 227 pp.

A History of Vector Analysis by Michael J. Crowe. University of Notre Dame Press, Notre Dame, 1967. xvii + 270 pp.

The Development of the Foundations of Mathematical Analysis from Euler to Riemann by I. Grattan-Guinness. MIT Press, Cambridge, 1970. xiii + 186 pp.

Die Genesis des abstrakten Gruppenbegriffes by Hans Wussing. VEB Deutscher Verlag der Wissenschaften, Berlin, 1969. 258 pp.

Most mathematics has been developed since 1800, and most history of mathematics deals with the period before 1800. We can only guess at the