There is one other consideration to be brought up: cost. My copy of Cassels-Fröhlich cost $16.00 (before discounts); *Corps Locaux* lists for 36 F (roughly $6.60, plus shipping costs, etc.). That means that for a dollar more than the cost of Weiss' book you can have all of class field theory thrown in, and that anyone willing to read French can get more mathematics than in Weiss for about half the price. Anyone who buys Weiss will need some justification other than cost-effectiveness.

LAWRENCE CORWIN

Studies in Geometry, by Leonard M. Blumenthal and Karl Menger. W. H. Freeman and Co., San Francisco, 1970. 512 pp.

The trend towards abstraction and generalization in modern mathematics at times seems to be at odds with the need for an intuitive insight into which concepts will prove fruitful and significant to the further development of the body of mathematics. In *Studies in Geometry* we find geometrical intuition at its best treated from an abstract point of view, which allows for significant generalization—notably the development of metric and topological properties of Boolean algebras and the development of projective geometry of any finite dimension without the introduction of new definitions for elements of each dimension.

Studies in Geometry is neither a systematic development of one particular geometry nor is it a survey of the main topics of geometry. Rather it presents essentially three theories—the theory of distance geometry, culminating with metric characterizations of Banach and Euclidean spaces, the theory of projective and related geometries, and the theory of curves, considered both from a lattice-theoretical viewpoint and from the traditional three-dimensional Euclidean point of view. These three topics seem quite unrelated and one might expect to find little continuity in the plan of the book. This, however, is not the case. For these theories are all developed from the common concepts of set and lattice, which provide an underlying unifying element to their study.

While both authors use the same algebraic structures in their studies, it is apparent that there is a definite difference between their basic ideas of what geometry is. Blumenthal takes a somewhat Kleinian point of view—a geometry is a system $\{S, E\}$ where S is a set and E an equivalence relation defined in the set of all subsets, called figures, of S. Fundamental, therefore, to the study of a geometry is a study of the properties shared by all the figures of an equivalence class. To Menger, however, a geometry cannot be divorced from the fundamental operations of join and meet and hence his postulational systems consist of laws satisfied by these operations.

Parts 1 and 3 contain the work of Blumenthal and students which was developed over the last two decades and heretofore not published as a whole. Part 1 serves as an introduction to the basic theory of geometric and metric lattices, which culminates in an exposition of the theory of Boolean distance geometry. A Boolean metric space is a Boolean algebra B together with a mapping, called its distance function, of $B\times B$ into B satisfying the characteristic properties of a metric (where $x\leqq y$ means $xy=x$). Of primary importance is the distance function $d(a, b)=ab'+a'b$—it is unique among distance functions in that it imposes a group structure on B, a characteristic which I found noteworthy and elegant. With the Kleinian program in mind, Blumenthal gives a detailed study of distance preserving mappings, defining two figures to be congruent if there is a one-to-one distance preserving transformation of one onto the other. The fruitfulness of this study is underlined by the following theorem: if B_1 and B_2 are Boolean metric spaces such that each three points of B_1 are congruent with three points of B_2, then B_1 is congruent with a subset of B_2.

The latter half of Part 1 and the beginning of Part 3 develop the theory of betweenness and linearity in Boolean metric spaces. This development plays an essential role in the metric characterizations of Banach and Euclidean spaces, one of the major objectives of Part 3. These characterizations are quite charming so, with the reader's permission, I give the postulates here. A set S is a Banach space with unique metric lines provided a metric space M may be defined over S which is complete, convex, externally convex, and satisfies the two-triple property (if four distinct elements of S contain two triples that are congruent with triples of a Euclidean straight line, then the remaining two triples have that property also) and the Young property (involving distances of middle points of triples). In order to obtain a Euclidean space one adds one extra postulate, a weak Pythagorean assumption. Part 3 is completed with a purely metric study of the differential geometry of curves.

Part 2, the work of Menger and students, presents projective geometry as an algebra with two operations corresponding to join and meet. This theory makes its first appearance in its entirety in the present book and is based on exceedingly simple postulates. A projective structure is a set S with two binary operations $\cup$ and $\cap$ and containing two elements v and u satisfying the following laws:

(1) $v\cup x=x=u\cap x$, $v\cap x=v$, $u\cup x=u$ for any element x of S;

(2) $x\cup((x\cup y)\cap z)=x\cup((x\cup z)\cap y)$ and

$$x \cap ((x \cap y) \cup z) = x \cap ((x \cap z) \cup y)$$

for any three elements x, y, z of S.

By adjoining one assumption about the existence of a certain configuration, one obtains a complete foundation of finite-dimensional projective geometry.

The second half of Part 2 contains a treatment of two- and three-dimensional noneuclidean geometries, again developed solely in terms of join and meet. Of special interest in the development of hyperbolic geometry is the definition of betweenness: the point Q is said to be between P and R if for any two lines p and r on P and R, respectively, each line q on Q meets at least one of p or r. It is significant that while this definition may be formulated in projective or affine planes, the relation so defined in them lacks the basic properties of betweenness. The remainder of this section gives some new applications of fragments of the affine and hyperbolic planes to kinematics.

Part 4 deals with the theory of curves as it was essentially developed in the 1920's. It begins with a discussion of the various classical definitions of curves and the problems encountered and then proceeds with the definition used today, presenting the main theorems of the theory. By this method of presentation, Menger succeeds in giving one of the clearest demonstrations of the procedure of mathematical thought and the creation of a mathematical theory.

The authors make no assumptions about the reader's mathematical background, except the requirement of some mathematical maturity; proofs are given in their entirety and these features, combined with the many exercises, mark *Studies in Geometry* as an excellent reference for a graduate course in geometry. Moreover, because much of the material in this book is new or has never been published as a whole before, *Studies in Geometry* should prove highly interesting to the professional mathematician.

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