A THEOREM ON COVARIANT CONSTANT (1, 1)-TENSOR FIELDS

BY C. D. JEFFRIES

Communicated by H. S. M. Coxeter, April 13, 1970

This note announces the proof of the following

THEOREM. *Suppose M is an n-dimensional C^∞ manifold with a connection ∇ and a $(1, 1)$-tensor field J which satisfy $\nabla J = 0$. Then M admits a Riemann structure G and a second connection $\tilde{\nabla}$ satisfying $\tilde{\nabla} G = 0$ and $\tilde{\nabla} J = 0$.*

We have a proof which explicitly constructs such a G and $\tilde{\nabla}$ in terms of distributions on M defined by the semisimple and nilpotent parts of J. An alternative proof using the theory of principle fibre bundles [**1**] stems from showing that for any endomorphism $J: \mathbf{R}^n \to \mathbf{R}^n$, the centralizer of J in $\mathrm{GL}(n)$ is diffeomorphic to the manifold product of a space of orthogonal commutants of J (with respect to a certain inner product) and a Euclidean space. The decomposition was initially derived using matrix methods, but Dr. S. Halperin has suggested a shorter proof of the decomposition using a theorem of K. Iwasawa [**2**].

Implicit in the theorem is that the adjoint J^* of J with respect to G satisfies $\tilde{\nabla} J^* = 0$, or equivalently, that J^* commutes with the specified orthogonal commutants of J. In the special case that J is an almost tangent structure ($J^2 = 0$, $2 \cdot \operatorname{rank} J = n$), it turns out that $\mathcal{J}_1 = J + J^*$ and $\mathcal{J}_2 = J - J^*$ satisfy $\mathcal{J}_1^2 = -\mathcal{J}_2^2 = I$ and $\mathcal{J}_1 \mathcal{J}_2 = -\mathcal{J}_2 \mathcal{J}_1$. By first choosing a Riemann structure, C. S. Houh [**3**] was able to derive the relations for $\mathcal{J}_1$ and $\mathcal{J}_2$ and thereby the $\tilde{\nabla}$ of the theorem. Also studying almost tangent structures, H. Wakakuwa and S. Hashimoto [**4**] were able to specify an inner product on $\mathbf{R}^n$ and derive using matrices the above decomposition of the centralizer of J in $\mathrm{GL}(n)$. They further noted the commutativity of J^* (and hence $\mathcal{J}_1$ and $\mathcal{J}_2$) with the orthogonal commutants of J.

Another application of the theorem shows that if M admits a cyclic J with real eigenvalues, then M admits a flat connection. D. E. Blair and A. P. Stone [**5**] observed this fact with the stronger assumptions that ∇ is a Riemann connection and that J has n distinct real eigenvalues.

AMS 1969 *subject classifications*. Primary 5350, 5352; Secondary 1530, 1538.

Key words and phrases. *G*-structures, connections, *O*-deformable, tensor fields, commuting matrices.

References

1. S. Kobayashi and K. Nomizu, *Foundations of differential geometry*, Vol. 1, Interscience, New York, 1963. MR **27** #2945.

2. K. Iwasawa, *On some types of topological groups*, Ann. of Math. (2) **50** (1949), 507–558. MR **10**, 679.

3. C. S. Houh, *On a Riemann manifold M_{2n} with an almost tangent structure*, Canad. Math. Bull. **12** (1969), 759–769.

4. H. Wakakuwa and S. Hashimoto, *Remark on almost tangent structure*, Tensor **20** (1969), 270–272.

5. D. E. Blair and A. P. Stone, *A note on the holonomy group of manifolds with certain structures*, Proc. Amer. Math. Soc. **21** (1969), 73–76. MR **38** #5133.

University of Toronto, Toronto, Ontario, Canada