# TWO NEW $H$-SPACES

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It is the purpose of this note to announce the following result.

THEOREM. (i) *The total space of any principal* $SU(3)$ *bundle over* $S^7$ *is an H-space.*

(ii) *There are exactly four homotopy types of such total spaces.*

Two of these homotopy types are known $H$-spaces; namely, $SU(3)\times S^7$ and $SU(4)$. The other two are the new $H$-spaces of the title, and a word is in order as to in what sense they are new.

If one seeks differentiable manifolds which are $H$-spaces not homeomorphic to known $H$-spaces, then recent work of Belfi [1] and Morgan [4] furnish a big supply. For example, there are infinitely many nonhomeomorphic manifolds having the homotopy type of $SU(4)$ (and hence being $H$-spaces). If one seeks new homotopy types (excluding, of course, cartesian products of known ones) the picture is quite different. Classically one knew only $S^7$ and its projective space $P^7$, except for Lie groups. In 1968 Hilton and Roitberg [2], [3] discovered a new $H$-space, a principal $S^3$ bundle over $S^7$. In 1969 Stasheff [5] found two more new $H$-spaces among the seven homotopy types of principal $S^3$ bundles over $S^7$. Our two new spaces brings the known total to seven in dimension $\leqq 15$.[3] We have also shown that the three new homotopy types introduced by going from principal $S^3$ bundles over $S^7$ to $SO(4)$ 3-sphere bundles over $S^7$ are not $H$-spaces.

The first part of our theorem is proved using the technique of mixing homotopy types (relative to a subdivision of the set of prime numbers) due to Zabrodsky [7], in much the same manner as Stasheff [5]. The second part uses the Adams operations in $K$-theory and a result of Suter [6] to distinguish the homotopy types.

## REFERENCES

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[3] By using [7] and a theorem of W. Browder, Zabrodsky gets infinitely many H-manifolds in higher dimensions. Recently Roitberg has used [7] to obtain some new 14-dimensional H-manifolds.

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