THE NONEXISTENCE OF BRANCH POINTS IN THE CLASSICAL SOLUTION OF PLATEAU'S PROBLEM

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The result to be proved is the following.

THEOREM. *Let C be an arbitrary Jordan curve in R^3. Denote by Δ the closed unit disk in R^2. Then there exists a regular minimal surface S of the type of the disk spanning C. Specifically, there exists a map*

$$h: \Delta \rightarrow R^3$$

satisfying

(i) *h is continuous in Δ;*

(ii) *h maps the boundary of Δ homeomorphically onto C;*

(iii) *each component h_k of h is a harmonic function in the interior of Δ, hence the real part of an analytic function Φ_k. The functions Φ_k satisfy*

$$\sum_{k=1}^{3} (\Phi_k')^2 \equiv 0 \tag{1}$$

and

$$\sum_{k=1}^{3} |\Phi_k'|^2 \neq 0 \quad \textit{everywhere.} \tag{2}$$

(iv) *the surface S defined by h has least area among all maps of Δ into R^3 satisfying* (i) *and* (ii); *if the area of S is infinite, then every interior portion of S has minimum area with respect to its own boundary curve.*

It is well known that condition (iii) implies that h is a conformal immersion of the interior of Δ onto a regular minimal surface. (See for example [**1**, §II.18].)

The above theorem was proved by Douglas and Radó (see [**1**, §§VI.1–7]) *except* for condition (2). Since the functions Φ are analytic and not all constant by (ii), it follows that condition (2) can fail at most at isolated points. Such points are called *branch points*. It has remained an open question whether these branch points actually

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occur. We show that they do not. Specifically, what we prove is the following.

THEOREM. *Any mapping h which satisfies all the conditions of the above theorem except for* (2) *must also satisfy* (2).

The proof depends on a close analysis of the behavior of a minimal surface in the neighborhood of a branch point. The following property is used.

LEMMA. *Let $h(w)$ define a minimal surface having a branch point at $w=0$. Then either*

(a) *there exists $\delta>0$ such that if $0<|w_1|<\delta$, $0<|w_2|<\delta$ and $h(w_1)=h(w_2)$, then the tangent planes to h at w_1 and w_2 are distinct*; *or*

(b) *there exists a local parameter $\tilde{w}$ in a neighborhood of $w=0$ such that h is invariant under some rotation about $\tilde{w}=0$.*

Suppose that a solution surface h had a branch point. We may assume that it lies at the origin. Condition (b) of the lemma would contradict property (ii). But using (a) we can replace h in a neighborhood of $w=0$ by a mapping having the same boundary values but strictly smaller area, contradicting (iv).

Details will appear elsewhere.

REFERENCE

1. T. Radó, *On the problem of Plateau*, Vol. 2, Ergebnisse der Mathematik und ihrer Grenzgebiete, Springer-Verlag, Berlin, 1933.

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