product in older books, are supposed to be known. For this, as well as for some of the details left out, the reader might find useful the book *Structure of rings* by N. Jacobson, which is a standard reference, but it is also more general and more difficult to read. There are several misprints in the book such as exponents which appear as factors, some changed symbols, a couple of exponents omitted, etc. We think that only two of the misprints have some importance. One which could cause some trouble when only the theorem is consulted appears in the statement of Theorem 5.1.7 where instead of $\sum_{i=1}^{k}\chi_i(a)\chi_i(b)=0$, it should read $\sum_{i=1}^{k}\chi_i(a)\overline{\chi_i(b)}=0$. In Theorem 6.3.2 the ring B is a direct product (or complete direct sum) of fields, and not just a direct sum as stated.

This book will appeal to many a reader. It would be wonderful as a textbook, and, in fact, it is based on the author's lecture notes published by the University of Chicago. But it can also be useful as a reference and as a source of information on counterexamples and recent literature. Only people looking for the most general form of a particular theorem are advised to turn to other books, but the reader interested in studying or reviewing its subject-matter or looking for a rounded account of it could do no better than choosing this book for this purpose.

MARIA J. WONENBURGER

Introduction to the theory of abstract algebras, by R. S. Pierce. Holt, New York, 1968. ix+148 pp. $5.50.

If the author of one book on universal algebra is asked to review another, he inevitably thinks in terms of comparisons. This is especially so when these are the only two books published in the field (though a third, by G. Grätzer, will appear in 1969). There should be no harm in making such comparisons as long as one guards against bias, both rational (preferring one's own children because one considers them more beautiful) and irrational (preferring them because they are one's own flesh and blood). The reviewer has made an earnest attempt to excise all such bias from this review; any traces that remain will easily be spotted by the reader, who has thus been forewarned.

Professor Pierce's book is intended as an introduction for graduate students, and clearly not necessarily students specializing in algebra, for as the author rightly says: "familiarity with this theory should be standard equipment for all mathematicians." After two introductory chapters, one reviewing the necessary set-theory and one on the basic concepts (homomorphisms, subalgebras, congruences, etc.) the author takes four basic theorems of universal algebra and devotes a chapter to each. They are

(a) the subdirect decomposition theorem (expressing an algebra as a subdirect product of subdirectly irreducible algebras),

(b) direct decompositions and the Krull-Schmidt theorem (in the version of Ore),

(c) the existence of free products, and

(d) Birkhoff's characterization of varieties of algebras.

The author stresses the lattice aspect of the subject ("the monograph might appropriately be subtitled 'An introduction to the theory and use of lattices'"). This is also the reviewer's opinion and in this respect the two books have a similar emphasis. However, Pierce's treatment is more general in two ways: he admits partial algebras and infinitary operations. Both of these can be justified by their applications, especially outside algebra itself, and it seems worthwhile to put up with the slight extra complication of definitions and proofs. However, the added generality of stating the definitions of homomorphism, subalgebra, etc. for relational systems rather than algebras does not really pay off, as the author himself remarks. It seems that relational systems are too different, even from partial algebras, to be treated in the same way. Paradoxically, an exercise in Chapter 1 shows how to regard relational systems as a special case of partial algebras and this strengthens the case for either giving a separate treatment or (as the author essentially does, after Chapter 1) omitting relational systems altogether.

The subdirect decomposition theorem for finitary partial algebras is proved by showing that the lattice of congruences is compactly generated, and proving that in a compactly generated lattice every element is a meet of meet-irreducible elements. This would seem a natural place to explain the precise connexion of compactly generated lattices to lattices of subalgebras of finitary algebras, rather than the exercise to which it is relegated by the author. For the Krull-Schmidt theorem the algebras have to be full but not finitary, and the theorem is again proved first for lattices. The development includes the Zassenhaus, Schreier and Jordan-Hölder theorems for modular lattices (but gives no clue to their connexion with the picture on the cover). Some interesting results on direct decompositions of lattices are given in the exercises.

With the chapter on free algebras the substratum changes from lattice-theory to category-theory, but the latter remains far more in the background; in fact not much more than the definition is given. Again this is a point of view with which the reviewer concurs: the correspondence between universal algebra and category-theory is not at present close enough to warrant a common development (it would be of interest to give such a development, which at the same time preserves the intuitive simplicity of the universal algebra approach). The basic theorem of this chapter takes a class $\mathfrak{A}$ of algebras closed under subalgebras and products, a family (A_i) of $\mathfrak{A}$-algebras which can all be embedded in the same $\mathfrak{A}$-algebra and constructs the "free $\mathfrak{A}$-sum" (=free product) of the A_i. To the reviewer it would seem more natural to start from the universal construction of the coproduct (=free composition) which exists in a more general context and then find conditions for the canonical mappings to be injective. The case of partial algebras requires minor modifications which are the subject of an exercise; finitarity of operations is not assumed anywhere. The final chapter gives a careful definition of word algebras (requiring in general expressions of infinite length) and a proof that a class of algebras is a variety (i.e. equationally definable) if and only if it is closed under subalgebras, products and quotients. As an application (requiring infinitary operations) the author derives the theorem of Loomis-Sikorski that every countably complete Boolean algebra is σ-representable.

This brief outline shows that the book gives the novice a very solid grounding in the subject, including several of the most basic results. The many definitions necessary are carefully discussed and motivated; it is quite uncharacteristic that no distinction is made between families and sets. Occasionally one feels the need for an adjective, such as 'full,' to describe algebras that are *not* partial; thus Proposition 1.6.3 leaves a doubt in the reader's mind whether the 'algebras' are partial or full. Special mention should be made of the numerous exercises, ranging from trivial verifications to complete do-it-yourself kits for such topics as l-groups, complemented lattices, etc. Among the minor irritations of the book are uneconomic definitions. E.g. separate definitions are given of (i) upper bound (ii) least upper bound and (iii) least (in this order). It would seem more natural to define (i), (iii) and let (ii) look after itself. The same thing happens with lattices, which are actually defined after complete lattices. In the definition of an identity, '$v=w$' is described as a sequence; but unless the notion of '$=$' is formalized, this is not a sequence as defined in the book. The printing is clear, although the theorems throughout the book are set in roman type and for this reason are rather hard to pick out. But these are very minor criticisms and do not seriously detract from the enjoyment of this book, which is a welcome addition to the literature.

P. M. COHN