

of Jordan algebras will be found in the forthcoming book by Jacobson."

A final chapter presents some results on power-associative algebras. This class of algebras includes the Jordan and alternative algebras studied before.

As for the level of the presentation the author says: "I expect that any reader will be acquainted with the content of a beginning course in abstract algebra and linear algebra. Portions of six somewhat more advanced books are recommended for background reading, and at appropriate places reference is made to these books for results concerning quadratic forms, fields, associative algebras, and Lie algebras." The list of books is very good, but the portions which afford background material are in any case small and in some instances nil for the author only needs particular results mentioned there. The reader can forget this, for in the text reference is made to the page which, if need be, should be consulted. We think that the demands of the book for the conscientious reader come rather from the amount of identities which have to be established and the number of clever substitutions in, and manipulations of these identities. This seems unavoidable at the present state of the theory and we hope that this book will contribute to the finding of new results and more streamlined proofs.

The author seems to have been very careful in the preparation of the book. In spite of the amount of subindices and hundreds of equations which it includes, we have found only a couple of inessential misprints.

In our opinion the bibliography would have been more useful if at the end of each chapter there were an indication of the items which, besides the ones cited in the text, are connected with its content. This is only a very small objection to a book which contains very interesting results not available in other books; written in a plain and clear style it reads very smoothly if one is ready to skip the details of the proofs and the computations.

MARIA J. WONENBURGER

*Noncommutative rings*, by I. N. Herstein. The Carus Mathematical Monographs, no. 15, Mathematical Association of America, 1968. xi+199 pp. \$6.00.

This very beautiful book is the result of the author's wide and deep knowledge of the subject-matter combined with his gift for exposition.

The well-selected material is offered in an integrated presentation

of the structure theory of noncommutative (associative) rings and its applications. Besides seeing the theory at work in the study of groups of matrices, group representations, and in settling the problems of Burnside and Kurosh, there are, or there are given the bases for the construction of, counterexamples, so the reader can see how a theorem fails under weaker assumptions, he is also informed of open questions and of the latest generalizations of some theorems and some theories. Each chapter ends with a choice list of references; hence the interested reader can go to the original papers to study the generalizations not presented in the book.

The style is lively and smooth. Definitions are kept to a minimum and the statements of the theorems are sharp and clear. Although there are not lists of exercises one can check his understanding of the material and try his hand at the theory by supplying a few proofs, some details of other proofs or of the counterexamples which are left to the reader. This could be the more difficult point for the less experienced mathematician, since the proofs although neat are condensed and at times it is tacitly assumed that the reader can see for himself the points on which the proof rests.

The author is more interested in giving a transparent presentation of the theory as a whole and of the theorems in particular, than in giving an account of their most general versions. Since the central theme is the structure theory of associative rings, these are not assumed to possess an identity element. On the other hand the definitions and theorems refer to rings and the book does not try, when possible, to present some of the definitions and theorems for modules, and afterwards to specialize to rings by considering a ring as a module relative to the regular representation. Following this line, the density theorem is given only for primitive rings, since completely reducible (semisimple) modules are not even defined. But from this density theorem, as the author points out, "the whole structure theory flows." It is unnecessary to add that the book has a classical flavor and is almost entirely untouched by homological algebra.

Let us now try to describe in general lines the architecture of the book, without trying to do justice to the wealth of results included.

The book starts with a study of the Jacobson radical and some general theorems on semisimple Artinian rings. Then, in Chapter 2, the density theorem is proved and applied to the structure theory. The chapter also states the general and bounded Burnside problems for groups, and, as an application of the Wedderburn-Artin Theorem, shows that the answer is affirmative in the case of groups of matrices.

The basic part of the structure theory is included in these two

chapters and forms the background for the rest of the book, whose remaining chapters are independent of each other. Later on, in Chapter 7, the structure theory is complemented with the "simple, short and neat proof" of the Goldie's theorems due to Procesi and Small, although "it is not nearly as revealing as the proof given earlier by Procesi." The more special theory of central simple algebras is covered in Chapter 4, which, besides the discussion of the Brauer group and crossed products, contains a section suitably entitled "Some classic theorems", where among others the Noether-Skolem, the double centralizer and the Jacobson-Noether theorems are proved, as well as the theorem on the derivations of central simple finite dimensional algebras. The proof that every element of the Brauer group has finite order is given as a consequence of this property for the cohomology groups of order  $n > 0$  of finite groups. This is the only place in the book where homological algebra appears briefly.

The usefulness of the structure theory in the study of commutativity theorems, that is, in showing that "appropriately conditioned rings are commutative" is shown in Chapter 3. The proofs of such theorems follow a general pattern amply illustrated in the results contained in the book. The other topic about which the general structure theory yields important results is the study of rings satisfying polynomial identities (Chapter 6). These offer the appropriate frame for the statement of Kurosh's problem and the results afford an affirmative answer to the problem for the case of algebraic algebras of bounded degree.

As for the applications of the theory of rings to group theory, Chapter 5 on the representations of finite groups ends up with the famous classical theorems of Burnside and Frobenius. Before giving these applications of the theory of characters, the author remarks: "It is difficult to see or to explain why this machinery when turned loose on a group works so effectively, but work indeed it does." The content of this chapter which was one of the pillars for the development of the theory of algebras sets out the scope of its applications.

A short final chapter offers part of the recent work (1964) of Golod and Shafarevich and its applications in showing that the answer to the general Burnside and Kurosh problems is in the negative.

For the reader with no more background than a "good first course in algebra" as described by the author in the preface, he indicates where to find the basic material assumed as known in some parts of the book and with which the reader may not feel familiar. The basic properties of the tensor product of algebras, also called Kronecker

product in older books, are supposed to be known. For this, as well as for some of the details left out, the reader might find useful the book *Structure of rings* by N. Jacobson, which is a standard reference, but it is also more general and more difficult to read. There are several misprints in the book such as exponents which appear as factors, some changed symbols, a couple of exponents omitted, etc. We think that only two of the misprints have some importance. One which could cause some trouble when only the theorem is consulted appears in the statement of Theorem 5.1.7 where instead of  $\sum_{i=1}^k \chi_i(a)\chi_i(b) = 0$ , it should read  $\sum_{i=1}^k \chi_i(a)\overline{\chi_i(b)} = 0$ . In Theorem 6.3.2 the ring  $B$  is a direct product (or complete direct sum) of fields, and not just a direct sum as stated.

This book will appeal to many a reader. It would be wonderful as a textbook, and, in fact, it is based on the author's lecture notes published by the University of Chicago. But it can also be useful as a reference and as a source of information on counterexamples and recent literature. Only people looking for the most general form of a particular theorem are advised to turn to other books, but the reader interested in studying or reviewing its subject-matter or looking for a rounded account of it could do no better than choosing this book for this purpose.

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*Introduction to the theory of abstract algebras*, by R. S. Pierce. Holt, New York, 1968. ix+148 pp. \$5.50.

If the author of one book on universal algebra is asked to review another, he inevitably thinks in terms of comparisons. This is especially so when these are the only two books published in the field (though a third, by G. Grätzer, will appear in 1969). There should be no harm in making such comparisons as long as one guards against bias, both rational (preferring one's own children because one considers them more beautiful) and irrational (preferring them because they are one's own flesh and blood). The reviewer has made an earnest attempt to excise all such bias from this review; any traces that remain will easily be spotted by the reader, who has thus been forewarned.

Professor Pierce's book is intended as an introduction for graduate students, and clearly not necessarily students specializing in algebra, for as the author rightly says: "familiarity with this theory should be standard equipment for all mathematicians." After two introductory chapters, one reviewing the necessary set-theory and one on the basic concepts (homomorphisms, subalgebras, congruences, etc.) the