CONNECTIVE FIBERINGS OVER BU AND U

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Eilenberg and Moore [1] have developed a spectral sequence converging to the cohomology of the total space of an induced fibration. L. Smith [2] has recently developed methods by which this spectral sequence can be computed in the special case of a fibration induced by an H-map from the pathspace fibration over a $K(Z, n)$. Using these methods, we have computed the cohomology rings $H^*(BU(2n, \cdots, \infty), Z_p)$ and $H^*(U(2n+1, \cdots, \infty), Z_p)$, p an arbitrary prime, thus extending the work of Adams [3] and Stong [4]. (We use the symbol $X(n, \cdots, \infty)$ to denote the $n-1$ connective fibering over a space X.)

If M is a graded Z_p-module, denote by $F(M)$ the free Z_p-algebra generated by M. Let $\mathrm{Op}(\beta P^1 i_n)$ denote the sub-Hopf algebra of $H^*(K(Z, n)Z_p)$ generated over the Steenrod algebra by the single element $\beta P^1 i_n$, and define graded Z_p-modules M_n in such a way that $F(M_n) = \mathrm{Op}(\beta P^1 i_n)$. Finally, if n is an integer it can be written uniquely in the form $n = a_0 + a_1 p + \cdots + a_k p^k$, with $a_i < p$. Set $\sigma_p(n) = \sum a_i$.

THEOREM. *Let p be an odd prime. There exist indecomposable cohomology classes $\theta_{2i} \in H^{2i}(BU, Z_p)$ such that*

$$(1)\qquad H^*(BU(2n, \cdots, \infty), Z_p) = \frac{H^*(BU, Z_p)}{Z_p[\theta_{2i} \mid \sigma_p(i-1) < n-1]} \otimes \prod_{t=0}^{p-2} F[M_{2n-3-2t}],$$

$$(2)\qquad H^*(U(2n+1, \cdots, \infty), Z_p) = \frac{H^*(U, Z_p)}{E[\mu_{2i+1} \mid \sigma_p(i) < n]} \otimes \prod_{t=0}^{p-2} \left\{ F[M_{2n-2-2t}] \otimes E\left[\nu_{2ip^k+1} \,\Big|_{k>0}^{\sigma_p(i-1)=n-t-2}\right] \right\}$$

as tensor products of Hopf algebras.

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[1] These results are contained in the author's Ph.D. thesis done at Princeton University in 1967 under the direction of J. C. Moore. It is a pleasure to thank Dr. Moore for his help and encouragement. Advice on the use of the spectral sequence came from Larry Smith. I have had helpful conversations with R. Stong.

Our methods also permit determination of the cohomology of each stage of the Postnikov tower of U

$$H^*(U(1, \cdots, 2n-1), Z_p) = E[\mu_{2i+1} | \sigma_p(i) < n] \otimes \prod_{t=0}^{p-2} F[M_{2n-1-2t}] \tag{3}$$

as a tensor product of Hopf algebras.

One can write down results corresponding to (1), (2) and (3) for the case $p=2$. These agree with the work of Stong [4], and with the unpublished work of Miss Vastersavendts.

These results with coefficients in Z_p permit us to determine divisibility conditions on integral cohomology classes. If m is a real number greater than zero, let $\operatorname{lig}(m)$ denote the least integer greater than or equal to m; if $m \leqq 0$, set $\operatorname{lig}(m)=0$. Let $c_k \in H^{2k}(BU, Z)$ be the Chern class, and let $r_n \colon BU(2n, \cdots, \infty) \to BU$ be the canonical projection.

THEOREM. *$r_n^*(c_k)$ is divisible by*

$$\prod_p p^{\operatorname{lig}[((n-1)-\sigma_p(k-1))/(p-1)]}$$

and by no greater number.

In the stable range $k < 2n$ this divisibility condition agrees with the stable result of Adams [3].

The proofs of these results will appear elsewhere.

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4. R. E. Stong, *Determination of $H^*(BO(k, \cdots, \infty)Z_2$ and $H^*(BU(k, \cdots, \infty)$ $\cdot Z_2)$*, Trans. Amer. Math. Soc. **107** (1963), 526–544.

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