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INSTITUTE FOR ADVANCED STUDY

AUTOMORPHISM GROUPS OF FINITELY GENERATED NILPOTENT GROUPS

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It is rare for any property of a group G to carry over to its automorphism group. Recently J. Lewin [1] constructed a finitely presented group whose automorphism group is not even finitely generated. Now finitely generated nilpotent groups are finitely presented (see e.g. [2]). So Lewin's example contrasts strikingly with the following.

THEOREM A. *The automorphism group of a finitely generated nilpotent group is finitely presented.*

In a way Theorem A reinforces the commonly held view that the automorphism group of a finitely generated nilpotent group is, from a group-theoretical viewpoint, quite simple. Now Philip Hall [3] has proved that a finitely generated nilpotent group has a faithful representation in $\text{GL}(n, \mathbb{Z})$, the integer unimodular group of degree n . So the following generalization of Hall's theorem might be thought of as another indication of the controlled nature of finitely generated nilpotent groups and their automorphism groups.

THEOREM B. *The holomorph of a finitely generated nilpotent group (i.e. the split extension of the group by its automorphism group) has a faithful representation in $\text{GL}(n, \mathbb{Z})$ for some n .*

The proofs of Theorem A and Theorem B use general Lie-theoretic techniques and a result which is of independent interest, namely

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Theorem C below. In order to state Theorem C we recall that two groups are commensurable if they are both finite extensions of a common group.

THEOREM C. *The automorphism groups of commensurable finitely generated nilpotent groups are commensurable.*

It seems likely that these theorems have analogues for polycyclic groups. These analogues and complete proofs of Theorems A, B, and C will appear elsewhere.

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