A CONSTRUCTION FOR MAXIMAL (+1, −1)-MATRIX OF ORDER 54[1]

BY C. H. YANG

Communicated by J. D. Swift, November 12, 1965

Let m_n be an $n \times n$ matrix with entries +1 or −1 and α_n be the maximal absolute value of det (m_n). When $n \equiv 2 \pmod 4$, it is known that

$$\alpha_n^2 \leqq 4(n-2)^{n-2}(n-1)^2 = \mu_n \qquad \text{(see Ehlich [1])}$$

and

$$\alpha_n = \mu_n^{1/2}, \quad \text{for } n \leqq 50, \text{ except } n = 22 \text{ and } 34 \quad \text{(see [1], [2])}.$$

It is found that the above equality also holds for $n = 54$.

The maximal (+1, −1)-matrix M of order 54 can be constructed as follows:

$$M = \begin{pmatrix} A_1 & A_2 \\ -A_2^T & A_1^T \end{pmatrix},$$

where A_1, A_2 are circulant matrices of order 27 and T indicates the transposed matrix. The first rows of A_1 and A_2 are found respectively as:

A_1: − + − + − + + − + + + + + + + + − + + − − + + + + − −

A_2: − + + + + + − + − + − + + − + + − + + + − − + + − − −

where − means −1 and + stands for +1. The absolute value of det(M) can be obtained easily from

$$MM^T = \begin{pmatrix} P & 0 \\ 0 & P \end{pmatrix} \quad \text{where} \quad P = \begin{bmatrix} 54 & & 2 \\ & \ddots & \\ 2 & & 54 \end{bmatrix}.$$

References

1. H. Ehlich, *Determinantenabschätzungen für binäre Matrizen*, Math. Z. **83** (1964), 123–132.

2. C. H. Yang, *Some designs for maximal* (+1, −1)*-determinant of order* $n \equiv 2$ (mod 4), Math. Comp. **20** (1966), 147–148.

State University of New York, College at Oneonta

[1] This research was supported by Summer Research Fellowship of The Research Foundation of State University of New York.