baffled many who thought themselves familiar with homotopy theory. Among its prescriptions, it asks us "Write, following (1.2a), $I^{m+n} = I^m \times I_m^{m+n}, \cdots$." (1.2a) turns out to be (with the misprint corrected)

$$X^{I^n}, A^{\dot{I}^n}, x_0^{J^{n-1}} = X^{I^k \times I_k^n}, A^{\dot{I}^k \times I_k^n \cup I^k \times \dot{I}_k^n}, x_0^{J^{n-1}}.$$

It was some time before the reviewer realized that "following (1.2a)" refers to what *follows* (1.2a) and not (1.2a) itself!

Misprints are excessively numerous. Of particular importance are the following. There are two on p. 7. First the explanation of the symbol a_i^j is flatly contradicted by the instance given; second we have the unenlightening statement that the 0 matrix $\mathbf{0}$ is the square matrix $\mathbf{0} = (0)$. Then on p. 201 the meaning of Definition 4.2 is obscured by having $\langle \boldsymbol{h} \otimes \boldsymbol{g} \rangle$ instead of $\langle h \otimes g \rangle$. On p. 215 a delicate point is lost because in a discussion involving ψ and Ψ, many ψ's appear as Ψ. On p. 412 we can only assume that Lemma 6.4 is a misprint, although we have not reconstructed it (perhaps it would assert that A is a deformation retract of $\Omega(A, Z)$); and on pp. 415, 416, amid other misprints, there occur some highly confusing replacements of X, Y by x, y. The bibliography misspells the names Eckmann, Hirzebruch and Wylie; and the reference to Hopf's paper on group-manifolds has evidently been through an unusually efficient scrambler.

To sum up, the reviewer admires the sweep and coverage achieved by the author; he and the author would have chosen differently from the supply of special topics to illumine the basic material, but that is surely no criticism. The reviewer would have preferred less "general topology" to make room for the cohomology topics listed at the start of this review, but this is just a matter of taste. The reviewer's real disquiet springs from his feeling that the text before him is not yet thoroughly ready for publication and requires substantial emendation and editing along the lines indicated. He trusts that his criticisms may be interpreted in this constructive light and that a new and greatly improved edition of this book may appear.

PETER HILTON

The foundations of intuitionistic mathematics. By Stephen Cole Kleene and Richard Eugene Vesley.

This book consists of four chapters, three by Kleene and one by Vesley. The authors' general purpose is to formalize a portion of intuitionistic analysis and to pursue certain investigations within and certain investigations about the formal system. Such an enterprise, however admirable the mathematics involved, may not be sympathetic to the constructively-minded mathematician, who perhaps is historically justified if he believes it will contribute to misleading the mathematical community as to the methods and philosophy of constructivism. Indeed it seems to be the common practice in the logical literature, and this book is no exception, to speak of intuitionistic mathematics as if large parts of it are identifiable with the workings of this or that formal system. In this regard not all mathematicians will share the belief implicitly expressed in Chapter I that the formal reasonings involved in Heyting's formalization of intuitionistic number theory, of which the author's system is an extension, lead to constructively valid results because they can be performed in terms of the meaning. For instance it can be and has been argued that the use of implication in Heyting's system is idealistic and not in accord with any fixed and precise interpretation.

Chapter I, written by Kleene, extends Heyting's formal system, as expounded in Kleene's book I. M. (Introduction to metamathematics) by adding additional symbols and axioms. The new symbols are function variables (variables for sequences of natural numbers), certain function symbols, and Church's λ. The first group of new axioms, concerning quantification with respect to function variables, offers no surprises. Neither do the two following axioms, which concern equality and the λ operator. Next comes an axiom

$$\forall x \exists \alpha A(x, \alpha) \supset \exists \alpha x \forall (x, \lambda y \alpha(2^x \cdot 3^y))$$

of more interest, which might be described as a selection principle or an axiom of choice. The point is that a choice function *should* exist constructively, since a selection is implicit in the very meaning of existence. The next group of new axioms consists of recursion equations for the function symbols f_i, whose specific forms are left open in order to permit the incorporation into the system of whatever special functions are desired. In this way the exponential function and some other special functions are added to the system, following which a generous number of theorems and meta-theorems are obtained.

The final two new axioms, which attach themselves to Brouwer's theory of spreads, are the most interesting. Brouwer's theory of spreads and the machinery of free choice sequences which it entails are notoriously hard to understand. The author's discussion is lucid and precise. His analysis results in founding the theory on two axioms, the Bar Theorem and Brouwer's principle. The Bar Theorem is a classically valid statement that asserts that any property R applicable to finite sequences of natural numbers which is enjoyed by some initial segment of every sequence of natural numbers must have a certain form. Brouwer's principle roughly stated says that if f is any function from the set of natural number sequences to the natural numbers then to every natural number sequence $\{\alpha_n\} = \alpha$ corresponds an integer N such that $f(\beta) = f(\alpha)$ whenever $\alpha_1 = \beta_1, \cdots, \alpha_N = \beta_N$. This is classically false.

Possibly this treatment of the theory of spreads, replacing as it does an unsatisfactory proof by a pair of axioms, should be preferred to Brouwer's own. Of course the use of an axiomatic treatment means that the theory of spreads can't be part of constructive mathematics, but this should disturb only those constructivists who believe the theory fills a genuine mathematical need.

The fact that Brouwer's principle contradicts classical mathematics might lead a classical mathematician to question the consistency of the formal system. This problem is settled in Chapter II, written by Kleene, by an interpretation of the formal system in terms of recursive function theory which, it is asserted, is based on only principles acceptable classically as well as intuitionistically. This is the notion of *realizability*, expounded in Kleene's book I. M. and expanded and modified here. The author believes the semantical arguments using this interpretation could be formalized to give a metamathematical consistency proof relative to the formal system minus Brouwer's principle.

In Chapter III Vesley develops a portion of analysis within the formal system. Due to the absence of the principle of the excluded middle even the elementary notions offer complications which have no classical analog. Brouwer's principle and the Bar Theorem are used to show that every real-valued function on the interval $[0, 1]$ is uniformly continuous, a famous result of Brouwer. Other properties of the continuum are developed, including a rather weak form of compactness for $[0, 1]$. Many of the proofs seem quite formidable. It is surprising that compactness in the usual sense was not proved, since such a result, properly interpreted, would seem to be well within the reach of the formal system.

The last chapter, by Kleene, investigates further the complicated situation which obtains relative to the order and inequality properties. Certain assertions of Brouwer about these relations are shown to be valid in the formal system whereas certain others are not.

The book contains an impressive quantity of original mathematics. In fact it is a collection of four, interrelated research papers. The writing is lucid, very much in the style of I. M. While it can be examined with profit by anyone who has read I. M. it should prove especially valuable to those whose interests lie in the formalism and philosophy of intuitionistic mathematics.

ERRETT BISHOP