RESEARCH PROBLEMS

6. E. T. Parker: *Finite combinatorial set theory.*

Let a, b, c be positive integers, $a > b > c$. Given a set S of a elements and a class K of n distinct subsets of b elements each from S, there must exist three distinct sets in K having at least c elements in common, provided n is sufficiently large. Develop upper and lower bounds on $n(a, b, c)$. (The *three* sets can of course be generalized to $t > 1$.) (Received January 30, 1964.)

7. G. B. Dantzig: *Eight unsolved problems from mathematical programming.*

a. Let C_n be an n-dimensional bounded polyhedral convex set defined by $2n$ distinct faces, n of which determine the extreme point p_1 and the remaining n of which determine the extreme point p_2. Does there always exist a chain of edges joining p_1 to p_2 such that the number of edges in the chain is n?

b. Let E be the extreme points of a unit n-cube having as faces the coordinate hyperplanes through the origin and the hyperplanes parallel to them passing through the point $(1, 1, \cdots, 1)$. Let P be a given hyperplane which separates E into two parts E_1 and E_2. Characterize the $n-1$ dimensional faces of the convex set having E_1 as its complete set of extreme points.

c. A matrix such that the determinant of every square submatrix has value -1, 0, or $+1$ is called *unimodular*. The distinct columns of such a matrix are said to form a *complete set* if the annexation of a column not in the set destroys the unimodular property. Two such sets *belong to the same class* if one can be obtained from the other by a permutation of the rows of its matrix. Characterize the various classes. How can they be generated? Given a matrix, find necessary and sufficient conditions that it satisfy the unimodular property.

d. Given an $n \times n$ permutation matrix $[x_{ij}]$, i.e., a zero-one matrix each row and column of which have exactly one unit element. Let the set of its n^2 elements constitute a point in n^2-dimensional coordinate space. It is known that these and only these points are extreme points of the convex set defined by

$$\sum_{i=1}^{n} x_{ij} = 1, \qquad j = 1, 2, \cdots, n,$$

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad i = 1, 2, \cdots, n, \qquad x_{ij} \geqq 0.$$

It is not known, however, how to characterize the faces of the convex polyhedral sets whose extreme points consist only of those permutation matrices which represent n-cycles.

e. If a square matrix M has k nonzero elements, k multiplications and additions are required to multiply M by an arbitrary vector b. Its inverse M^{-1}, which is assumed to exist, may have more than k nonzero elements, hence may require more than k of each of these operations if multiplied by an arbitrary vector b. However, M^{-1} may be represented (in fact in many ways) as a product of elementary matrices generated by row and column transformations on M. Prove or disprove the conjecture that there always exists a representation such that carrying out the indicated operations with b will require no more than $l=k$ multiplications and $l'=k$ additions. If not true, find sharp lower bounds for l and l'.

f. Given a class of n-dimensional simplices sharing in common $n-1$ of the hyperplanes defining $n-1$ of their faces such that the remaining face of each simplex is parallel to a given hyperplane. Find the largest simplex in the class the interior of which contains no lattice point.

g. Geometrically the simplex method moves from one vertex to the next neighboring vertex which gives the greatest change in the value of a linear form (where variables correspond to the coordinates of a point). For convex sets defined by m equations in n nonnegative variables the number of such moves is remarkably low (often between m and n in practice). Intuitively, wandering on the outside of a convex set in this manner would appear to be extremely inefficient, yet empirical evidence from thousands of cases is to the contrary. Why?

h. Usually there is an analogue for linear inequality systems of a theorem about linear equality systems. The problem of minimizing a positive definite quadratic form subject to linear equation constraints is reducible (by the method of Lagrange) to solving a certain system of linear equations. If the constraints are replaced by linear inequalities, is the entire problem reducible to that of solving a certain system of linear inequalities (i.e., to an equivalent linear program)? (Received February 23, 1964.)

8. Ralph De Marr: *Extension of commuting mappings.*

First problem. Let X be a topological space having the property that if X_0 is a nonempty closed subset of X and $f_0\colon X_0\to X$, where f_0 is continuous, then f_0 can always be extended to a continuous mapping $f\colon X\to X$. If $f_0\colon X_0\to X_0$ and $g_0\colon X_0\to X_0$, where f_0 and g_0 are continuous and $f_0(g_0(x))=g_0(f_0(x))$ for all $x\in X_0$, then is it possible to extend both f_0 and g_0 to continuous mappings $f: X \to X$ and $g: X \to X$ such that $f(g(x)) = g(f(x))$ for all $x \in X$?

Second problem. Let X be a complete lattice. It is known that if X_0 is a nonempty subset of X and $f_0: X_0 \to X$, where f_0 is isotone (i.e., $f_0(x) \leqq f_0(y)$ if $x \leqq y$), then f_0 can be extended to an isotone mapping $f: X \to X$. If $f_0: X_0 \to X_0$ and $g_0: X_0 \to X_0$, where f_0 and g_0 are isotone and $f_0(g_0(x)) = g_0(f_0(x))$ for all $x \in X_0$, then is it possible to extend both f_0 and g_0 to isotone mappings $f: X \to X$ and $g: X \to X$ such that $f(g(x)) = g(f(x))$ for all $x \in X$?

Nothing is known even in the case where $X = [0, 1]$. (Received March 16, 1964.)

THE APRIL MEETING IN RENO

The six hundred tenth meeting of the American Mathematical Society was held on Saturday, April 18, 1964 at the University of Nevada in Reno, Nevada. There were 100 registrants at this meeting, 90 of whom were members of the Society.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, hour addresses were given by Professor George B. Dantzig of the University of California at Berkeley, and by Professor Marshall Hall, Jr. of the California Institute of Technology. Professor Dantzig spoke on *Mathematics of the decision sciences.* The title of Professor Hall's talk was *Block designs.* Professor E. M. Beesley introduced Professor Dantzig, and Professor J. L. Selfridge presented Professor Hall.

There were four sessions for contributed papers with Professors T. J. McMinn, W. H. Simons, R. F. Tate, and L. E. Ward, Jr. presiding.

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