Computations are made for the cyclic groups and some results about the transfer homomorphism obtained.

Chapter 6 deals with the reduced pth powers for odd primes p. Results analogous to those of the first two chapters are proved for the Steenrod algebra $\mathcal{A}(p)$ and applications given, including the result that the p-primary component of $\pi_i(S^3)$ is zero for $i<2p$ and Z_p for $i=2p$.

The next two chapters are devoted to the construction of the reduced powers, the verification of the axioms and the uniqueness theorem. Briefly, the construction is as follows. Let K be a complex, π a subgroup of the symmetric group $\mathcal{S}(n)$ on n letters, and W a π-free acyclic complex. If $K^n=K\times K\times \cdots \times K$ (n factors), π acts on $W\times K^n$ by the diagonal action and we denote the quotient by $W\times_\pi K^n$. Now, if u is a cohomology class on K and $u^n=u\times \cdots \times u$ the n-fold external product of u on K^n, we can, under suitable circumstances, extend u^n in a natural way to a class Pu on $W\times_\pi K^n$. Letting $d\colon K\rightarrow K^n$ be the diagonal map, we have a map $1\times_\pi d\colon W\times_\pi K = W/\pi\times K\rightarrow W\times_\pi K^n$ and an induced map $(1\times_\pi d)^*\colon H^*(W/\pi\times K)\rightarrow H^*(W\times_\pi K^n)$. If the coefficient domain is a field, $H^*(W/\pi\times K)\approx H^*(W/\pi)\otimes H^*(K)$ and we define the set of reduced powers of u to be the elements u_i of $H^*(K)$ where $(1\times_\pi d)^*Pu=\sum v_i\otimes u_i \in H^*(W/\pi)\otimes H^*(K)$. In particular, the Steenrod reduced pth powers are obtained when π is the cyclic group of $\mathcal{S}(p)$ generated by the unique p-cycle.

The concluding chapter of the book is an appendix by D. B. A. Epstein in which purely algebraic derivations are given for some properties of the Steenrod algebra which previously had mixed geometric-algebraic derivations.

The book is extremely well written and the choice of material excellent. The prerequisites have been kept at a minimum so that only a first course in algebraic topology is required. It certainly would be a valuable addition to the library of any topologist and, in fact, to any library already having the introductory books in algebraic topology.

R. H. Szczarba

Differential forms with applications to the physical sciences. (Mathematics in science and engineering, vol. 11, ed. by R. Bellman.) By Harley Flanders. Academic Press, New York, 1963. 12+203 pp. $7.50.

This is a remarkable book, presenting the fundamental ideas of the geometry of manifolds in a robust, unpedantic and clear manner; in fact, it is like no other modern mathematics text, since it treats relatively advanced material in an uninhibited spirit, very similar to that of good theoretical physics.

Of course, differential geometry and theoretical physics were once very close; the recent divergence has been particularly harmful to geometry. Physical scientists are stuck with a working view of geometry essentially dating from, say, 1925, when the "débauch d'indices" was at its height, hence do not know how to phrase their problems so as to interest mathematicians. On the other side, the geometric parts of mathematics seem beset by sterility and formalism for lack of real problems, which is sadly ironic, because geometry, by its very nature, resists formalization, and requires highly developed intuitive powers very similar to those of a physicist. The appearance of this book at this time is very fortunate, since there are signs that the situation is changing. Physicists and theoretically-minded engineers seem to be faced with the fact that the traditional mathematics that they know well (indeed, better than most mathematicians) is failing them, and that they must look for new tools. Some pure mathematicians (still a minority, however) are beginning to appreciate again the rich structure of mathematical physics and classical geometry and the wealth of fascinating problems they pose.

As the title indicates, the technical foundation of the book is the differential form calculus of Elie Cartan. Chapter 1 is a brief introduction to differential forms as things inside integrals. Some comments on the relation to tensor analysis are so good that they should be quoted, especially since they give an idea of Flanders' prose and style.

"Tensor analysis per se seems to consist only of techniques for calculations with indexed quantities. It lacks a body of substantial or deep results established once and for all within the subject and then available for application. The exterior calculus does have such a body of results. If one takes a close look at Riemannian geometry as it is customarily developed by tensor methods one must seriously ask whether the geometric results cannot be obtained more cheaply by other machinery. In classical tensor analysis, one never knows what is the range of applicability simply because one is never told what the space is. . . . Tensor fields do not behave themselves under mappings. . . . With exterior forms we have a really attractive situation in this regard. If $\phi\colon M\rightarrow N$, and if ω is a p-form on N, there is naturally induced a p-form $\phi^*(\omega)$ on M. . . . In tensor calculations the maze of indices often makes one lose sight of the very great differences between various types of quantities which can be represented by tensors. . . . It is often quite difficult using tensor methods to discover the deeper invariants in geometric and physical situations, even the local ones. Using exterior forms, they seem to come naturally according to these principles: (i) All local geometric relations arise one way or another from the equality of mixed partials, i.e. Poincaré's lemma. (ii) Local invariants themselves usually appear as the result of applying exterior differentiation to everything in sight. (iii) Global relations arise from integration by parts, i.e. Stokes' theorem. (iv) Existence problems which are not genuine partial differential equations generally are of the type of Frobenius-Cartan-Kähler system of exterior differential forms and can be reduced thereby to systems of ordinary equations." (This is a bit overenthusiastic; the "Cartan-Kähler" part really involves partial differential equations, namely Cauchy-Kowalewski systems, but Flanders is in a higher sense correct since the approach via forms is much better suited than the standard methods to dealing with the geometric problems involving partial differential equations.) "In studying geometry by tensor methods, one is inevitably restricted to the natural frames associated with a local coordinate system. . . . Now who in his right mind would study Euclidean geometry with oblique coordinates? . . . We are led to introduce moving frames, a method which goes hand-in-glove with exterior forms."

Chapters 2 and 3 give the main properties of differential forms in Euclidean spaces. Chapter 4 exploits this with several classical applications to Euclidean three space, beautifully done using moving frames. It is a joy to see the theory of curvature of surfaces done side-by-side with Maxwell's equations! Chapter 5 extends to manifolds, and defines integration of forms over smooth singular simplices. The de Rham theorems are then stated, motivating a touch of algebraic topology. Chapter 6 contains brief applications, such as the definition of the degree of a mapping by an integral and the Hopf invariant.

Chapter 7 is "Applications to differential equations." Among the topics treated (very neatly) are the Poisson and Green formulas, uniqueness for the heat equation, and, in greater detail, the Frobenius integration theorem, together with some of its geometric applications. For example, one finds Cartan's little-known proof that a Lie algebra generates a Lie group germ. Chapter 8, "Applications to differential geometry," begins with hypersurfaces in Euclidean spaces, and goes on to give a taste of Riemannian geometry and affine connections. Flanders is so skillful that he is almost convincing that the relation

"$dP=\omega_i e_i$" means something! At any rate, he does present the Christoffel symbols and curvature tensor using, as always, differential forms in a clever and consistent way. Chapter 9 is "Applications to (Lie) group theory," discussed briefly from the form point of view with no mention of Lie algebra.

The closing Chapter 10 presents standard material from classical point and fluid mechanics. The treatment is more-or-less equivalent to that in a good physics book; here is a point where greater geometric sophistication would have paid off in terms of greater understanding and clarification. However, it is done as cleanly and simply as one could wish, and should serve well as a basis if material in greater depth is attempted in a course.

Another remarkable feature of the book is the wide clientele it can serve, ranging from its main goal of introducing physical scientists to differential forms to a thorough second year graduate course in geometry. Flanders puts it so well that we must quote him again: "Our goal is to develop an intuition and a working knowledge of the subject with as much dispatch as is possible. . . . This falls short of the extremely great precision which is customary in modern abstract mathematics and pretty much inherent in its nature. One who quite rightly is searching recent developments in mathematics for applicable material must find this precision a considerable barricade, overpedantic if not downright tedious—a very real factor in the great separation between modern mathematics and modern science. . . . In spite of all this, we do not hesitate to recommend this material to graduate students in mathematics as an introduction to modern differential geometry . . . . Considering the degree to which present day mathematical training consists of one abstraction after another, some of the things in this book could be a bit of an eye-opener, even to a mathematics student who is well along. For example, one could envisage such a student meeting here a parabolic differential equation, or a matrix group, or a contact transformation for the very first time." Of course, Flanders' passion for simplicity has forced him to slight the well-rounded view of geometry that is essential in a systematic graduate course. By a happy coincidence, this book very nicely complements the recent one by Auslander and Mackenzie, and the two together should give the student an introduction to the works of Cartan, and such a book as Helgason's, which are at the heart of modern geometry. The reviewer hopes that all the universities in the country will incorporate the pedagogical principles inherent in the book into their geometry courses, and will start interdisciplinary courses to exploit the territory that has been opened up by Flanders. In the reviewer's opinion many of the graduate courses in geometry now are too oriented towards only presenting the machinery and hence have been a dismal failure; the students who are not geometrically oriented usually learn little real geometry and cannot handle the simplest computations with ease, while those who go on in geometry must often painfully learn the intuitive background for themselves later on. For example, it would be very reasonable to consider knowledge of this book as constituting the geometry part of the Ph.D. qualifying exam, especially since it is suited to self-study.

I can only finish my praise by remarking that we have heard much talk recently about the need for "interdisciplinary studies" and "applied mathematics," but Flanders seems to be one of the few who has actually done it; we owe him our thanks.

ROBERT HERMANN

Ensembles parfaits et séries trigonométriques. By Jean-Pierre Kahane and Raphael Salem. Acualités Scientifiques et Industrielles, No. 1301. Hermann, Paris, 1963. 192 pp. 27 F.

Algebraic numbers and Fourier analysis. By Raphael Salem. D. C. Heath and Co., Boston, 1963. 68 pp. $3.00.

The book by Kahane and Salem is a very welcome and stimulating addition to the literature on Fourier analysis. Very little of the material it contains has previously been published in book form. Although the subject matter may appear to be rather special, the study of these topics has shed light on problems of general harmonic analysis, such as the structure of the convolution algebra of the Borel measures on a locally compact abelian group, or the problem of spectral synthesis. Serious students of harmonic analysis should be particularly attracted by the many and varied techniques which are skillfully exhibited by the authors.

In Chapter 1 certain classes of perfect sets on the line (or the unit circle) are described which play a role later. For the purpose of this review, let us confine our attention to the simplest class, namely to the "symmetric perfect sets" defined as follows: Let $\xi_1, \xi_2, \xi_3, \cdots$ be positive numbers, less than 1/2, put

$$r_k = \xi_1\xi_2 \cdots \xi_{k-1}(1 - \xi_k),$$

and let E be the set of all x of the form

$$x = \epsilon_1 r_1 + \epsilon_2 r_2 + \epsilon_3 r_3 + \cdots$$

where $\epsilon_i = 0$ or 1; E is a perfect, totally disconnected set in [0, 1]. If $\xi_i = \xi$ for all i, the resulting set E is said to be "of constant ratio of