connecting the fixed points of T. (4) If O_1 and O_2 are disjoint simply connected domains invariant under a loxodromic T, the corresponding arcs, as in (3), divide S into two Jordan regions, one or the other of which must contain any domain disjoint from O_1 and O_2. (5) If O is a simply connected domain invariant under an elliptic T, then O must contain a fixed point of T.

The examples are elaborations of the ideas in L. R. Ford, *Automorphic functions*, 2nd ed., Chelsea, 1951, pp. 55–59.

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DIFFERENTIABLE NORMS IN BANACH SPACES[1]

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1. **Introduction.** In [4, p. 28] S. Lang has asked whether or not a separable Banach space has an admissible norm of class C^1. In this note we indicate a proof of the following theorem, which characterizes those Banach spaces for which such a norm exists.

THEOREM 1. *A separable Banach space has an admissible norm of class C^1 if and only if its dual is separable.*

It follows from this theorem that not even $C(I)$ possesses an admissible differentiable norm.

2. **Preliminaries.** Let X be a Banach space with norm α; we write $S_\alpha = \{x \mid \alpha(x) = 1\}$ and $B_\alpha = \{x \mid \alpha(x) \leqq 1\}$. A norm in X is admissible if it induces the same topology as does α. The dual space is written X^* and the norm dual to α is denoted by α^*. An $f \in X^*$ is called a support functional to B_α at $x \in S_\alpha$ if $\alpha^*(f) = f \cdot x$; if f has norm 1, it is called a normalized support functional and is written ν_x. A norm is smooth if there is a unique normalized support functional to B_α at each $x \in S_\alpha$. The norm α is differentiable at $x \neq 0$ if there is an $\alpha'(x) \in X^*$ such that

$$\lim_{y \to x; y \neq x} \frac{|\alpha(y) - \alpha(x) - \alpha'(x) \cdot (y - x)|}{\alpha(y - x)} = 0$$

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and a norm differentiable at each $x \in X - \{0\}$ is of class C^1 if the map $\alpha': X - \{0\} \to X^*$, defined by $x \to \alpha'(x)$, is continuous. The following two results are well known:

1. Klee [3]. Let X and X^* be separable. Then there exists an admissible norm α in X such that α^* is strictly convex, and such that whenever a sequence $\{f_n\}$ in X^* converges to $f \in X^*$ in the w^*-topology, then $\alpha^*(f_n) \to \alpha^*(f)$ implies $\alpha^*(f - f_n) \to 0$.

2. Bishop-Phelps [1]. In any Banach space X, the set of all the support functionals to B_α is dense in X^*.

3. **Proof of Theorem 1.** It is not difficult to see that if the norm α is differentiable at $x \in S_\alpha$, then $\alpha'(x) = \nu_x$ is a normalized support functional to B_α at x, and is unique. The map $x \to \nu_x$ of S_α into S_{α^*} is denoted by μ. We first establish the following general theorem:

THEOREM 2. (a) *If α is a smooth norm in X, then the map μ is continuous when the norm topology is used in X and the w^*-topology is used in X^*.*

(b) *The norm α is of class C^1 if and only if the map μ is continuous in the norm topologies.*

(c) *A norm is of class C^1 if and only if it is differentiable at every point of S_α.*

Complete details will be published elsewhere; using this result, we prove Theorem 1 as follows:

Assume X^* is separable, and let α be the norm of Klee's theorem. By a well-known duality, α is smooth. Theorem 2(a) assures μ is continuous with the w^*-topology in X^*, and then Klee's theorem shows μ is continuous in the norm topology. By Theorem 2(b), α is therefore of class C^1.

Assume now that α is of class C^1. Extend the continuous map μ to a continuous $\hat{\mu}: X - \{0\} \to X^*$ by setting $\hat{\mu}(x) = \alpha(x) \cdot \mu(x/\alpha(x))$. The image of $\hat{\mu}$ evidently contains the set of all the support functionals to B_α, and an application of the Bishop-Phelps theorem shows at once that X^* is separable whenever X is separable.

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