

by H. Flanders, in a course at the first year graduate level that would be of great interest to mathematically inclined physicists and engineers.

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Homology theory. An introduction to algebraic topology. By S. Wylie and P. J. Hilton. Cambridge University Press, New York, 1960. xv+484 pp. \$14.50.

The authors state their purpose and the intended scope of this book in the introduction as follows:

"This book has been written with the intention of providing an introduction to algebraic topology as it is practised today. The reader is not supposed at the outset to possess any knowledge of algebraic topology; indeed, even the reader with no knowledge of analytic topology or abstract algebra is provided, in the Background to Part I, with a synopsis of the facts that are taken for granted in the text. The treatment throughout has been subject to the consideration that, if the book is to serve its purpose, it must provide an account of the basic notions of algebraic topology intelligible to the mathematician inexperienced in the techniques and problems described. However, though the treatment is elementary, we have been more ambitious in our choice of material than is customary in elementary textbooks. It appears to us that the literature is rich in advanced textbooks and adequate in elementary introductory textbooks, but that the two types of book are not very effectively linked. Again, the advanced textbooks themselves fall into two classes which may broadly be described as classical and modern and the rapid shifts of emphasis which the subject has experienced make it difficult always to recognize classical arguments in their modern dress. We have tried to provide the links which we believe the student might find difficulty in providing for himself from a study of the available literature.

"Thus, while our beginning is quite elementary, we have been able, by omitting certain topics, particularly those treated canonically in classical works, to reach in later chapters the parts of the subjects which lie in the immediate foreground of present-day research."

In the opinion of the reviewer, they have been fairly successful in achieving this purpose. There is no other text treating algebraic topology that starts at the beginning and leads up to the topics of greatest interest in current research. On the one hand, there do exist the more advanced texts and specialized treatises dealing with special topics such as homotopy groups, fibre spaces, sheaves, manifolds, homological algebra etc., and on the other hand, there are the more

elementary books, such as those by E. M. Patterson, L. Pontrjagin, and A. H. Wallace which stop short of what is required to follow modern research on the subject.

Probably the main difficulty the reader of this book will experience, whether he is a novice in the subject or a professional topologist, is with the notation and terminology. First of all, there is much new and unconventional terminology and notation. The most radical innovation is the use of the terms *contrahomology*, *contrachain*, *contract*, etc. for the well-established terms *cohomology*, *cochain*, *co-cycle*, etc. Although it may be possible to defend this new terminology on strictly logical grounds, from a practical point of view it seems rather unfortunate. Such tampering with well-established notation tends to arouse strong emotions on the part of the practitioners of the subject and prevents the book from being used as much as it otherwise deserves. Other examples of unconventional notation are xf for $f(x)$ (the value of the function f at the point x), $C \cap G$ for $\text{Hom}(C, G)$, and $A \dagger B$ for $\text{Ext}(A, B)$.

In general, it seems best if authors use the same notation in a textbook on this level that they would use in a lecture to their professional colleagues.

In the second place, the authors seem to practice an undue proliferation of notation. For example although $H^*(X)$ is used (as usual) to denote the direct sum of the cohomology groups of X , when cup products are introduced the new notation $R^*(X)$ is used for the cohomology ring. As another example, in Chapter 8 entitled "Singular Homology Theory," the following symbols are introduced to denote various singular chain and cochain complexes on the space X :

$\Delta(X)$, $\tilde{\Delta}(X)$, $\Delta(X; G)$, $\Delta^{\cdot}(X; G)$, $\tilde{\Delta}(X; G)$, $\tilde{\Delta}^{\cdot}(X; G)$, $\Delta^b(X)$, $\Delta^N(X)$, $\square(X)$, $\square^b(X)$, $\square^N(X)$, $\tilde{\square}^N(X)$, and $\Delta^v(X)$; moreover, if X is the space of a simplicial complex K , there are also defined

$C^{\alpha}(K, G)$, $C_{\alpha}^{\cdot}(K, G)$, $C^{\omega}(K)$, $C_{\omega}^{\cdot}(K, G)$, $C^{\Omega}(K, G)$, and $C_{\Omega}^{\cdot}(K, G)$.

In addition, there are various chain and cochain complexes of pairs of spaces. The chain mapping induced on each of these complexes by a continuous or simplicial map also has a special notation.

There is no "Index of Symbols" at the end of the book; it is safe to say that if there were, the list would be quite long.

The book has many good features. Foremost among them in the reviewer's opinion is a wise choice of subject matter, especially with regard to its reference to present day research. In order to guide the reader, the authors have starred certain sections or parts of sections

which are somewhat more sophisticated than its place in the book would suggest and which may be omitted on first reading. Some material is set in small type to indicate that it is not central to the logical development of the subject. Each chapter ends with a collection of exercises, ranging from those which are easy or routine to those which are more difficult and interesting.

All mathematicians and students of mathematics who are interested in algebraic topology owe the authors a debt of gratitude for the production of this fine text.

W. S. MASSEY

Opere matematiche. Memorie e Note. By Vito Volterra. Vol. 1, xxxiii + 604 pp., 1954; vol. 2, 626 pp., 1956; vol. 3, 612 pp., 1957; vol. 4, 540 pp., 1960; vol. 5, 538 pp., 1962. Edited by the Accademia Nazionale dei Lincei, Rome. 40,000 Italian lire.

The Accademia Nazionale dei Lincei, in collaboration with the Consiglio Nazionale delle Ricerche, has successfully completed the enormous task of editing the mathematical papers of Vito Volterra (1860–1940) in the relatively short period of eight years from 1954 to 1962. The committee, always headed by the president of the Accademia, consisted originally of U. Amaldi, L. Amoroso, G. Armellini, U. D’Ancona, E. Freda, J. Pérès, E. Persico, M. Picone, A. Signorini, C. Somigliana and E. Volterra. For later volumes, B. Finzi and B. Segre very successfully carried the burden of the work left by the passing of Amaldi, Armellini, and Somigliana.

The mathematical community owes a debt of gratitude to the committee. It is a sad duty of the reviewer to mention here that after the completion of their work two more members, J. Pérès and A. Signorini, passed away.

The five volumes of these works contain most of Volterra’s mathematical papers, notes and memoirs. His well known books, some of which were written in collaboration with Pérès, Hostinski, and others, are of course not included in the present collection. Also omitted are some general addresses, obituaries, reviews, etc.

The first volume contains an address entitled “Vito Volterra” delivered by G. Castelnuovo at the inaugural meeting of the Accademia after its reconstitution in 1946. This is followed by a detailed analysis of the scientific work of the great mathematician, given by Somigliana. Finally there is a very complete biography of Volterra, written by his friend and collaborator, J. Pérès.

The papers are arranged in chronological order, beginning with a paper on the potential of an ellipsoid, published in 1881 before