RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

A REPRESENTATION THEOREM FOR STATIONARY MARKOV CHAINS

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Let $\{X_n\}$ be a real-valued strictly stationary stochastic process on the probability space $({}_0\Omega, {}_0\Sigma, {}_0P)$ and let $\{\xi_n\}$ be an independent sequence of random variables uniformly distributed on $[0, 1]$ for $n=0, \pm 1, \cdots$. The emphasis in [1], [2], [3], and [4] has been upon finding a function f such that the sequences $\{X_n\}$ and $\{f(\cdots, \xi_{n-1}, \xi_n)\}$ have the same probability structure (i.e., such that $X_{k_1}, \cdots, X_{k_n}$ and $f(\cdots, \xi_{k_1-1}, \xi_{k_1}), \cdots, f(\cdots, \xi_{k_n-1}, \xi_{k_n})$ have the same joint distribution for all positive integers n and all sequences $k_1, \cdots, k_n$). The sequence $\{f(\cdots, \xi_{n-1}, \xi_n)\}$ is considered to be just another "representation" of the original process $\{X_n\}$.

The theorem presented here gives a similar type of representation for *all* strictly stationary Markov processes with finite or denumerable state space.

Let ${}_0\Sigma_n$ be the σ-field of subsets of Ω generated by X_k for all $k \leqq n$ and let ${}_0\Sigma_{-\infty} = \bigcap {}_0\Sigma_n$. The σ-field ${}_0\Sigma_{-\infty}$ is called the tail field of the process $\{X_n\}$ and is said to be trivial if it contains only sets of probability zero and one. It has been shown (see [2], [3], and [4]) that if $\{X_n\}$ is a strictly stationary Markov process with a finite or denumerable state space then a necessary and sufficient condition for $\{X_n\}$ to have a one-sided representation $\{f(\cdots, \xi_{n-1}, \xi_n)\}$ is that $\{X_n\}$ be tail trivial.

Let $\{X_n\}$ be a strictly stationary Markov process with finite or denumerable state space. Let ${}_0T$ be the shift transformation induced on $({}_0\Sigma, {}_0P)$ by $\{X_n\}$ in such a way that $\{X_0 \in B\} = T\{X_1 \in B\}$, etc. The following theorem gives a representation for $\{X_n\}$ which depends on its tail field.

THEOREM (PART A). *There exists a probability space* $({}_1\Omega, {}_1\Sigma, {}_1P, {}_1T)$ *such that*

(1) ${}_1\Sigma$ *is the* σ*-field of all subsets of* ${}_1\Omega$,

(2) ${}_1P\{u\} > 0$ *for each* $u \in {}_1\Omega$,

(3) *$_1T$ is a measure preserving transformation from $_1\Omega$ to $_1\Omega$,*

(4) *$(_1\Sigma, {}_1P, {}_1T)$ and the tail field $(_0\Sigma_{-\infty}, {}_0P, {}_0T)$ of $\{X_n\}$ are isomorphic modulo sets of measure zero.*

Define the probability space $(_2\Omega, {}_2\Sigma, {}_2P, {}_2T)$ by

(1) $_2\Omega = \prod_{i=-\infty}^{\infty} [0, 1] = \{\xi = (\cdots, \xi_{-1}, \xi_0, \xi_1, \cdots) \mid \xi_i \in [0, 1]\}$,

(2) $_2\Sigma$ is the smallest σ-field of subsets of $_2\Omega$ with respect to which each coordinate projection ξ_i is Borel measurable,

(3) $_2P$ is product Lebesgue measure,

(4) $_2T\xi = (\cdots, \xi_0, \xi_1, \xi_2, \cdots)$.

Define $(\Omega, \Sigma, P, T) = (_1\Omega, {}_1\Sigma, {}_1P, {}_1T) \times (_2\Omega, {}_2\Sigma, {}_2P, {}_2T)$ and let Σ_0 be the smallest σ-field of subsets of Ω with respect to which the coordinate projections u and ξ_i for all $i \leqq 0$ are Borel measurable.

THEOREM (PART B). *There exists a function f measurable with respect to Σ_0 such that $\{f(T^n\xi)\}$ is a representation of $\{X_n\}$.*

Detailed proofs will appear elsewhere.

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