RESEARCH PROBLEMS

1. M. Slater: *Number theory.*

Research Problem 39 (Bull. Amer. Math. Soc. **68**, p. 557) may be sharpened as follows:

Can the integers 1 through n be paired with the integers $n+1$ through $2n$ so that no two of the $2n$ sums and differences $b_i \pm i$ are equal?

This is impossible for $n=2, 3, 6$; but I conjecture it holds for all other n. The problem may be regarded as a refinement of the problem of arranging n queens on an n by n chess board so that no two queens attack each other. Examples are

$n=4$: (1,7), (2,5), (3,8), (4,6).
$n=7$: (1,9), (2,14), (3,12), (4,10), (5,8), (6,13), (7,11).
$n=9$: (1,14), (2,18), (3,11), (4,13), (5,16), (6,12), (7,17), (8,15), (9,10).

(Received January 6, 1963.)

2. Richard Bellman: *Analysis.*

Let $u(z)$ be analytic in the region $|z| < a$, and consider the problem of determining the complex constants c_i, $i=0, 1, 2, \cdots, N$, so that

$$Q(c_1, c_2, \cdots, c_N) = \int_0^b \left| u^{(N)}(z) + c_1 u^{(N-1)}(z) + \cdots + c_N u(z) \right|^2 dz, \tag{1}$$

is a minimum, where $b < a$. Denote the minimum value by Q_N. Then:

(a) What is the asymptotic form of Q_N as $N \to \infty$?

(b) What are the asymptotic forms of the minimizing c_i?

Consider the more general problem where we wish to minimize

$$Q(c_0, c_1, \cdots, c_N) = \int_0^b \left| c_0 u^{(N)}(z) + c_1 u^{(N-1)}(z) + \cdots + c_N u(z) \right|^2 dz \tag{2}$$

over $\sum_{i=0}^N |c_i|^2 = 1$. What is the asymptotic form of the minimum value as $N \to \infty$?

The paper *On certain extremum problems for analytic functions*, by W. W. Rogosinski and H. S. Shapiro, Acta Math. **90** (1953), 287–318, contains background material which may be relevant. (Received December 20, 1962).